More on Meta-Stable Brane Configurations by Dualizing the Multiple Gauge Groups

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Abstract

We reexamine the $\mathcal{N}=1$ supersymmetric gauge theories with product gauge groups by adding the mass terms and the quartic terms for the flavors: two gauge group theory with fundamentals, bifundamentals and adjoints, three gauge group theory with fundamentals and bifundamentals, and their orientifold 4-plane generalizations. By moving the branes appropriately, we obtain the corresponding dual gauge theories. By analyzing the dual superpotentials, we present the type IIA nonsupersymmetric meta-stable brane configurations.

1 Introduction

The dynamical supersymmetry breaking in meta-stable vacua [1, 2] occurs in the $\mathcal{N} = 1$ supersymmetric gauge theory with massive fundamental quarks. Further quartic deformation for the quarks in the superpotential [3, 4] also has lead to the supersymmetry breaking meta-stable ground states when the gravitational attraction of NS5-brane [5] is considered. It is known, in the construction of supersymmetric ground states or its type IIA brane configurations [6], that a number of gauge theory duals(magnetic theory) involving product gauge groups can be interpreted in terms of branes of type IIA string theory. The question for the dynamical supersymmetry breaking in meta-stable vacua in the simplest $\mathcal{N} = 1$ supersymmetric product gauge theories which have both mass terms [1, 7, 8, 9] and quartic terms [3, 4] is answered recently in [10] when one dualizes the whole two gauge groups.

Although a single gauge group theory with two adjoint fields, where the superpotential has order $(k+1)$ and $(k'+1)$ for these adjoint fields, does not have dual theory correctly so far [11], the two gauge group theory with two adjoint fields as well as fundamentals and bifundamentals does have its dual description [12]. For $k = 2$ and $k' = 1$ case(i.e., two NS5-branes and one NS5'-brane), the meta-stable brane configuration for a single gauge group theory was found in [13] and for the $k = 1$ and $k' = 2$ case(i.e., one NS5-brane and two NS5'-branes), the meta-stable brane configuration for a single gauge group theory was studied in [14]. The type IIA brane configuration corresponding to two gauge group theory with fundamentals, bifundamentals and two adjoints was found in [15] some time ago and the brane configuration on triple product gauge group theory with fundamentals and bifundamentals was also studied by using the brane motion and linking number counting: the peculiar thing was the presence of full D4-branes ranging from $x^6 = -\infty$ to $x^6 = \infty$ in the brane configuration without changing the linking numbers.

In this paper, one reexamines these supersymmetric brane configurations [15] as well as their orientifold 4-plane generalizations and extracts the possible brane motions, during the dual process, for new meta-stable brane configurations, along the lines of [16, 17, 18, 19, 14, 10]. The common feature with the work of [10] and different feature with the previous works of [16, 17, 18, 19, 14] is that one dualizes the *whole product gauge groups*, not a single gauge group. The order of NS-branes is reversed completely after duality. The geometrical positions of the branes and the creation of D4-branes, during the dual process, play the important role for removing the unwanted gauge singlets [20, 21] and selecting the wanted gauge singlets. The magnetic theories we obtained are different from the one [15] in the sense that the dual color numbers are different and the dual superpotentials have different form: some of the

gauge singlets do not appear and we add the mass terms and quartic terms for the quarks. These deformation terms are realized geometrically in type IIA string theory.

In section 2, we review the type IIA brane configuration corresponding to the $\mathcal{N}=1$ $SU(N_c)\times SU(N_c')$ gauge theory with fundamentals, bifundamentals and *adjoints* and deform this theory by adding both the mass terms and the quartic terms for the fundamentals. The extra adjoint fields are realized by the multiple NS-branes in brane configuration. Then we describe the dual $\mathcal{N}=1$ $SU(\widetilde{N}_c)\times SU(\widetilde{N}_c')$ gauge theory with corresponding dual matter as well as the gauge singlets. We discuss the nonsupersymmetric meta-stable minimum by looking at the reduced dual superpotential and present the corresponding intersecting brane configuration of type IIA string theory.

In section 3, we review the type IIA brane configuration corresponding to the $\mathcal{N}=1$ $SO(2N_c)\times Sp(N_c')$ gauge theory with vectors, fundamentals, bifundamentals and *adjoints* and deform this theory by adding both the mass terms and the quartic terms for the vectors and fundamentals. Then we describe the dual $\mathcal{N}=1$ $SO(2\widetilde{N}_c)\times Sp(\widetilde{N}_c')$ gauge theory with corresponding dual matter as well as the gauge singlets. We describe the nonsupersymmetric meta-stable minimum and the corresponding intersecting brane configuration of type IIA string theory which is nothing but the brane configuration of section 3 with the addition of O4-plane.

In section 4, we review the type IIA brane configuration corresponding to the $\mathcal{N}=1$ $SU(N_c)\times SU(N_c')\times SU(N_c'')$ gauge theory with fundamentals, bifundamentals and deform this theory by adding both the mass terms and the quartic terms for the fundamentals. Then we describe the dual $\mathcal{N}=1$ $SU(\widetilde{N}_c)\times SU(\widetilde{N}_c')\times SU(\widetilde{N}_c'')$ gauge theory with corresponding dual matter as well as the gauge singlets. We discuss the nonsupersymmetric meta-stable minimum from the reduced dual superpotential and present the corresponding intersecting brane configuration of type IIA string theory.

In section 5, we review the type IIA brane configuration corresponding to the $\mathcal{N}=1$ $Sp(N_c)\times SO(2N_c')\times Sp(N_c'')$ gauge theory with fundamentals, vectors, bifundamentals and deform this theory by adding both the mass terms and the quartic terms for the fundamentals. Then we describe the dual $\mathcal{N}=1$ $Sp(\widetilde{N}_c)\times SO(2\widetilde{N}_c')\times Sp(\widetilde{N}_c'')$ gauge theory with corresponding dual matter as well as the gauge singlets. We discuss the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configuration of type IIA string theory which is nothing but the brane configuration of section 4 with the addition of O4-plane.

In section 6, we summarize the results of this paper and comment on the future directions.

2 $SU(N_c) \times SU(N_c')$ with N_f-, N_f'-fund., two adjoints, and bifund.

2.1 Electric theory

The type IIA supersymmetric electric brane configuration [15, 22, 23] corresponding to $\mathcal{N}=1$ $SU(N_c)\times SU(N_c')$ gauge theory [12] with N_f-fundamental flavors $Q, \widetilde{Q}$, N_f'-fundamental flavors $Q', \widetilde{Q}'$, bifundamentals $F, \widetilde{F}$ and two adjoint fields X_1, X_2 can be described as two middle NS5-branes, two left NS5'-branes, two right NS5'-branes, N_c- and N_c'-D4-branes, and N_f- and N_f'-D6-branes for the cubic superpotential of the adjoints. The X_1 is in the representation $(\mathbf{N_c^2}-\mathbf{1}, \mathbf{1})$ while the X_2 is in the representation $(\mathbf{1}, \mathbf{N_c'^2}-\mathbf{1})$, under the gauge group. The F is in the representation $(\mathbf{N_c}, \mathbf{N_c'})$ while the $\widetilde{F}$ is in the representation $(\overline{\mathbf{N_c}}, \overline{\mathbf{N_c'}})$, under the gauge group. The quarks Q and $\widetilde{Q}$ are in the representation $(\mathbf{N_c}, \mathbf{1})$ and $(\overline{\mathbf{N_c}}, \mathbf{1})$ respectively and the quarks Q' and $\widetilde{Q}'$ are in the representation $(\mathbf{1}, \mathbf{N_c'})$ and $(\mathbf{1}, \overline{\mathbf{N_c'}})$ respectively, under the gauge group.

The mass terms for each fundamental quarks can be added in the superpotential by displacing each D6-branes along

$$v \equiv x^4 + i x^5$$

direction leading to their coordinates $v = +v_{D6_{-\theta}} (+v_{D6_{-\theta'}})$ respectively while the quartic terms for each fundamental quarks can be added also by rotating each D6-branes [4] by an angle $-\theta(-\theta')$ in (w, v)-plane respectively. Here we define the complex coordinate w as

$$w \equiv x^8 + i x^9.$$

Then, the general superpotential from the one [12, 15, 23] by adding the above deformations is given by

$$\begin{aligned} W_{elec} &= \left[\frac{s_1}{3} \operatorname{tr} X_1^3 + \frac{s_2}{3} \operatorname{tr} X_2^3 + \operatorname{tr} X_1 \widetilde{F} F + \operatorname{tr} X_2 F \widetilde{F} + \lambda_1 Q X_1 \widetilde{Q} + \lambda_2 Q' X_2 \widetilde{Q}' \right] \\ &+ \frac{\alpha}{2} \operatorname{tr} (Q\widetilde{Q})^2 - m \operatorname{tr} Q\widetilde{Q} + \frac{\alpha'}{2} \operatorname{tr} (Q'\widetilde{Q}')^2 - m' \operatorname{tr} Q'\widetilde{Q}' \end{aligned} \tag{2.1}$$

where the parameters are described as the following geometric quantities

$$\alpha \equiv \frac{\tan\theta}{\Lambda}, \qquad \alpha' \equiv \frac{\tan\theta'}{\Lambda'}, \qquad m \equiv \frac{v_{D6_{-\theta}}}{2\pi \ell_s^2}, \qquad m' \equiv \frac{v_{D6_{-\theta'}}}{2\pi \ell_s^2}, \qquad \lambda_1 \equiv \sin\theta, \qquad \lambda_2 \equiv \sin\theta'.$$

The first two terms of (2.1), in general, are due to the rotation angles ω_L and ω_R of two left and right NS5'-branes with respect to the middle NS5-branes: $s_1 \equiv \tan \omega_L$ and $s_2 \equiv \tan \omega_R$.

We consider the case where $\omega_L = \omega_R = \frac{\pi}{2}$. Although the relative displacement of two color D4-branes, where the mass for the bifundamentals $m_F \equiv \frac{v_{NS5'}}{2\pi \ell_s^2}$ is the distance of D4-branes along the v-direction, can be added in the superpotential, we focus on the particular limit $m_F = 0$. Note that we also put the perturbations by $QX_1\widetilde{Q}$ and $Q'X_2\widetilde{Q}'$ in the superpotential [23] which will arise as the mesons in the magnetic theory. The lower order terms for the adjoints can occur when we displace the coincident NS5-branes and NS5'-branes in w direction and in v direction respectively.

Then the $\mathcal{N} = 1$ supersymmetric electric brane configuration for the superpotential (2.1) in type IIA string theory is given as follows and let us draw this brane structure in Figure 1 explicitly:

- Two middle NS5-branes in (012345) directions
- Two left NS5'-branes in (012389) directions
- Two right NS5'-branes in (012389) directions
- N_f $D6_{-\theta}$-branes in (01237) directions and two other directions in (v, w)-plane
- N_f' $D6_{-\theta'}$-branes in (01237) directions and two other directions in (v, w)-plane
- N_c- and N_c'-color D4-branes in (01236) directions

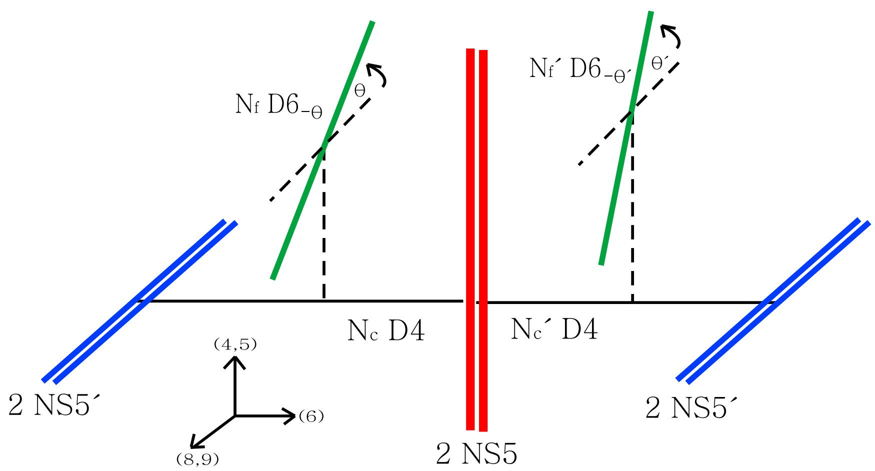

Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N_c')$ with bifundamentals $F, \widetilde{F}$, fundamentals $Q, \widetilde{Q}, Q', \widetilde{Q}'$, and adjoint fields X_1, X_2. The powers of X_1 and X_2 in the superpotential (2.1) are related to the number of NS-branes. A rotation of coincident $N_f(N_f')$ D6-branes in (w, v)-plane corresponds to a quartic term for the fundamentals $Q, \widetilde{Q}(Q', \widetilde{Q}')$ while a displacement of $N_f(N_f')$ D6-branes in $+v$ direction corresponds to a mass term for the fundamentals $Q, \widetilde{Q}(Q', \widetilde{Q}')$. Let us assume that $v_{D6_{-\theta}} < v_{D6_{-\theta'}}$ and $\theta < \theta'$.

2.2 Magnetic theory

For the meta-stable brane configurations, we should go to the magnetic theory corresponding to the previous electric theory. The left NS5'-branes start out with linking number $l_e = -\frac{2N_f'}{2} + N_c$ from Figure 1 and after duality these left NS5'-branes end up with linking number $l_m = \frac{2N_f'}{2} - \widetilde{N}_c' + 2N_f$ from Figure 2. In arriving at this, we consider the coincident NS-branes for the time being and assume that there is no splitting and reconnection between D4-branes in Figure 2. We consider only the particular brane motion where N_f $D6_{-\theta}$-branes meet the middle NS5-branes with *no angles*. That is, the $D6_{-\theta}$-branes become $D6_{-\frac{\pi}{2}}$-branes when they meet with the middle NS5-branes instantaneously and then after that they come back to the original $D6_{-\theta}$-branes [10]. Therefore, in this dual process, there is no creation of D4-branes. That is the reason for the $2N_f$ factor in the l_m, not $4N_f$. Then the dual color number $\widetilde{N}_c'$ is given by $\widetilde{N}_c' = 2N_f + 2N_f' - N_c$.

The right NS5'-branes start out with linking number $l_e = \frac{2N_f}{2} - N_c'$ and after duality these right NS5'-branes end up with linking number $l_m = -\frac{2N_f}{2} + \widetilde{N}_c - 2N_f'$. We consider only the particular brane motion where the $D6_{-\theta'}$-branes become $D6_{-\frac{\pi}{2}}$-branes when they meet with the middle NS5-branes instantaneously and after that they come back to the original $D6_{-\theta'}$-branes. Therefore, in this dual process, there is *no* creation of D4-branes. That is the reason for the $2N_f'$ factor in the l_m, not $4N_f'$. Then it turns out that the dual color number $\widetilde{N}_c$ is given by $\widetilde{N}_c = 2N_f' + 2N_f - N_c'$. Finally, one has the following dual color numbers

$$\widetilde{N}_c = 2N_f' + 2N_f - N_c', \qquad \widetilde{N}_c' = 2N_f + 2N_f' - N_c.$$

The low energy theory [12, 15] on the two color D4-branes has $SU(\widetilde{N}_c) \times SU(\widetilde{N}_c')$ gauge group and N_f-fundamental dual quarks $q', \widetilde{q}'$, N_f'-fundamental dual quarks $q, \widetilde{q}$, bifundamentals $f, \widetilde{f}$, two adjoints x_1, x_2 and various gauge singlets. The f is in the representation $(\widetilde{\mathbf{N}}_\mathbf{c}, \overline{\widetilde{\mathbf{N}}_\mathbf{c}'})$ while the $\widetilde{f}$ is in the representation $(\overline{\widetilde{\mathbf{N}}_\mathbf{c}}, \widetilde{\mathbf{N}}_\mathbf{c}')$, under the dual gauge group. The N_f' flavors q and $\widetilde{q}$ are in the representation $(\widetilde{\mathbf{N}}_\mathbf{c}, \mathbf{1})$ and $(\overline{\widetilde{\mathbf{N}}_\mathbf{c}}, \mathbf{1})$ respectively under the gauge group and in the representation $(\overline{\mathbf{N}_\mathbf{f}'}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{N}_\mathbf{f}')$ respectively under the flavor group $SU(N_f')_L \times SU(N_f')_R$. Similarly, the N_f flavors q' and $\widetilde{q}'$ are in the representation $(\mathbf{1}, \widetilde{\mathbf{N}}_\mathbf{c}')$ and $(\mathbf{1}, \overline{\widetilde{\mathbf{N}}_\mathbf{c}'})$ respectively under the gauge group and in the representation $(\overline{\mathbf{N}_\mathbf{f}}, \mathbf{1})$ and $(\mathbf{1}, \overline{\mathbf{N}_\mathbf{f}})$ respectively under the flavor group $SU(N_f)_L \times SU(N_f)_R$. In particular, a magnetic meson field $M_0 \equiv Q\widetilde{Q}$ is $N_f \times N_f$ matrix and comes from 4-4 strings of N_f flavor D4-branes, created from the intersection of $D6_{-\theta}$-branes and one of the right NS5'-branes, while a magnetic meson field $M_0' \equiv Q'\widetilde{Q}'$ is $N_f' \times N_f'$ matrix and comes from 4-4 strings of N_f' flavor D4-branes, created from the intersection of $D6_{-\theta'}$-branes and one of the left NS5'-branes. The adjoint fields x_1, x_2 correspond to the motion of two left and right NS5'-branes and two NS5-branes in (v, w)-plane.

Then the most general magnetic superpotential, when we consider the case where $N_f(N_f')$ $D6_{-\theta}$-branes($D6_{-\theta'}$-branes) meet the middle NS5-branes *with angles*, compared to the previous paragraph, is given by

$$
\begin{aligned}
W_{dual} &= \left[\frac{s_1}{3}x_1^3 + \frac{s_2}{3}x_2^3 + x_1\widetilde{f}f + x_2 f\widetilde{f} + \lambda_1 M_1 + \lambda_2 M_1'\right] \\
&+ \left(\frac{\alpha}{2}\,\mathrm{tr}\, M_0^2 - mM_0\right) + \left(\frac{\alpha'}{2}\,\mathrm{tr}\, M_0'^2 - m'M_0'\right) \\
&+ \left[M_0 q' x_2 f\widetilde{f}\widetilde{q}' + M_0' q x_1 \widetilde{f} f \widetilde{q} + M_1 q' f\widetilde{f}\widetilde{q}' + M_1' q \widetilde{f} f \widetilde{q}\right] \\
&+ \left[M_2\widetilde{q}' x_2 q' + M_3 \widetilde{q}' q' + M_2'\widetilde{q} x_1 q + M_3'\widetilde{q} q + P_1 q x_1 \widetilde{f}\widetilde{q}' + \widetilde{P}_1 q' x_2 f\widetilde{q} + P_2 q\widetilde{f}\widetilde{q}' + \widetilde{P}_2 q' f \widetilde{q}\right]
\end{aligned}
\tag{2.2}
$$

where the mesons are given by [12, 15]

$$
\begin{aligned}
M_0 &\equiv Q\widetilde{Q}, \quad M_0' \equiv Q'\widetilde{Q}', \quad M_1 \equiv QX_1\widetilde{Q}, \quad M_1' \equiv Q'X_2\widetilde{Q}', \\
M_2 &\equiv Q\widetilde{F}F\widetilde{Q}, \quad M_3 \equiv Q\widetilde{F}FX_1\widetilde{Q}, \quad M_2' \equiv Q'F\widetilde{F}\widetilde{Q}', \quad M_3' \equiv Q'F\widetilde{F}X_2\widetilde{Q}', \\
P_1 &\equiv Q\widetilde{F}\widetilde{Q}', \quad \widetilde{P}_1 \equiv Q'F\widetilde{Q}, \quad P_2 \equiv QX_1\widetilde{F}\widetilde{Q}', \quad \widetilde{P}_2 \equiv Q'X_2F\widetilde{Q}.
\end{aligned}
$$

The first two lines of (2.2) are dual expressions for the electric superpotential (2.1) and the corresponding meson fields M_0, M_0', M_1 and M_1' are replaced and the third and fourth lines of (2.2) are the analogs of the cubic term superpotential between the meson and dual quarks in Seiberg duality. Compared with the theory [24, 10] without two adjoints fields, there exist the extra meson fields coming from the adjoint fields X_1 and X_2:$M_1, M_1', M_3, M_3', P_2$ and $\widetilde{P}_2$.

Now we want to find out the reduced magnetic superpotential which is relevant to the meta-stable brane configuration we are interested in.

As observed in [10], when the N_f $D6_{-\theta}$-branes meet the middle NS5-branes, *no* creation of D4-branes implies that there is no M_2- or M_3-term in the above superpotential (2.2). The mesons M_2 and M_3 originate from $SU(N_c)$ chiral mesons $Q\widetilde{Q}$ when one dualizes the $SU(N_c)$ gauge group first by moving the middle NS5-branes to the left of the left NS5'-branes [25, 26, 27, 28, 29]. That is, the fluctuations of strings stretching between the $2N_f$ "flavor" D4-branes correspond to these meson fields. After two additional dual procedures, $SU(N_c')$ and $SU(\widetilde{n}_c)$, the cubic terms in the superpotential arise as M_2-dependent and M_3-dependent terms where M_2 has extra $\widetilde{F}F$ fields and M_3 has extra $\widetilde{F}FX_1$ fields, besides $Q\widetilde{Q}$, due to the further $SU(N_c')$-dualization. The M_2-term in the superpotential has an extra x_2 factor besides $\widetilde{q}'q'$.

Similarly, when the N_f' $D6_{-\theta'}$-branes meet the middle NS5-branes with *no angles*, there is no M_2'- or M_3'-term in the above superpotential (2.2). These meson fields M_2' and M_3' originate from $SU(N_c')$ chiral mesons $Q'\widetilde{Q}'$ when one dualizes the $SU(N_c')$ gauge group first by moving

the middle NS5-branes to the right of the right NS5'-branes. The strings stretching between the $2N_f'$ "flavor" D4-branes provide these mesons. After two additional dual procedures, $SU(N_c)$ and $SU(\widetilde{n}_c')$, the cubic terms in the superpotential arise as M_2'-term and M_3'-term where M_2' has extra $F\widetilde{F}$ fields and M_3' has extra $F\widetilde{F}X_2$ fields, besides $Q'\widetilde{Q}'$, due to the further $SU(N_c)$-dualization. The M_2'-term in the superpotential has an extra x_1 factor besides $\widetilde{q}q$.

Furthermore, when the N_f $D6_{-\theta}$-branes, the N_f' $D6_{-\theta'}$-branes and the middle NS5-branes meet each other with no angles, *no* P_1- and P_2- or $\widetilde{P}_1$- and $\widetilde{P}_2$-dependent terms arise in the superpotential (2.2). These mesons originate from $SU(N_c')$ chiral mesons $\widetilde{F}\widetilde{Q}'$ and FQ' when one dualizes the $SU(N_c')$ first by moving the middle NS5-branes to the right of the right NS5'-branes. The strings stretching between the $2N_f'$ flavor D4-branes and N_c color D4-branes give rise to these $2N_f'$ $SU(N_c)$ fundamentals and $2N_f'$ $SU(N_c)$ antifundamentals. After two additional dual procedures, $SU(N_c)$ and $SU(\widetilde{n}_c')$, these cubic terms arise as these meson terms where there exist extra $qx_1, \widetilde{q}x_2, q$ and $\widetilde{q}$ in the interactions of $P_1, \widetilde{P}_1, P_2$ and $\widetilde{P}_2$ in the superpotential respectively and these mesons have extra $Q, \widetilde{Q}, QX_1, \widetilde{Q}X_2$ fields respectively, due to the further $SU(N_c)$-dualization.

Then the reduced magnetic superpotential in our case by taking the first three lines of (2.2) is given by

$$\begin{aligned} W_{dual} &= \left[\frac{s_1}{3}x_1^3 + \frac{s_2}{3}x_2^3 + x_1\widetilde{f}f + \widetilde{f}x_2 f + M_1(q'f\widetilde{f}\widetilde{q}' + \lambda_1) + M_1'(q\widetilde{f}f\widetilde{q} + \lambda_2)\right] \\ &+ \left[M_0 q' x_2 f\widetilde{f}\widetilde{q}' + \frac{\alpha}{2}\,\mathrm{tr}\, M_0^2 - mM_0\right] + \left[M_0' q x_1\widetilde{f}f\widetilde{q} + \frac{\alpha'}{2}\,\mathrm{tr}\, {M_0'}^2 - m'M_0'\right]. \end{aligned} \tag{2.3}$$

Let us describe the meta-stable brane configuration with this magnetic superpotential. For the supersymmetric vacua, one can compute the F-term equations for this superpotential (2.3) and the F-terms for $M_0, q', \widetilde{q}', M_0', q, \widetilde{q}, f, \widetilde{f}, M_1, M_1', x_1$ and x_2 are given by

$$\begin{aligned} & q'x_2 f\widetilde{f}\widetilde{q}' - m + \alpha M_0 = 0, \qquad x_2 f\widetilde{f}\widetilde{q}' M_0 + f\widetilde{f}\widetilde{q}' M_1 = 0, \qquad (M_0 q' x_2 + M_1 q') f\widetilde{f} = 0, \\ & q x_1 \widetilde{f} f\widetilde{q} - m' + \alpha' M_0' = 0, \qquad x_1\widetilde{f}f\widetilde{q}M_0' + \widetilde{f}f\widetilde{q}M_1' = 0, \qquad (M_0' q x_1 + M_1' q)\widetilde{f}f = 0, \\ & (x_1\widetilde{f} + \widetilde{f}x_2) + \widetilde{f}\widetilde{q}'(M_1 q' + M_0 q' x_2) + \widetilde{q}(M_1' q + M_0' q x_1)\widetilde{f} = 0, \\ & (f x_1 + x_2 f) + \widetilde{q}'(M_1 q' + M_0 q' x_2) f + f\widetilde{q}(M_1' q + M_0' q x_1) = 0, \qquad q' f\widetilde{f}\widetilde{q}' + \lambda_1 = 0, \\ & q\widetilde{f}f\widetilde{q} + \lambda_2 = 0, \qquad s_1 x_1^2 + \widetilde{f}f + \widetilde{f}f\widetilde{q}M_0' q = 0, \qquad s_2 x_2^2 + f\widetilde{f} + f\widetilde{f}\widetilde{q}' M_0 q' = 0. \end{aligned} \tag{2.4}$$

The third, sixth, seventh and eighth equations of (2.4) are satified if the following equations hold

$$f x_1 = -x_2 f, \qquad x_1\widetilde{f} = -\widetilde{f}x_2, \qquad M_1 q' = -M_0 q' x_2, \qquad M_1' q = -M_0' q x_1. \tag{2.5}$$

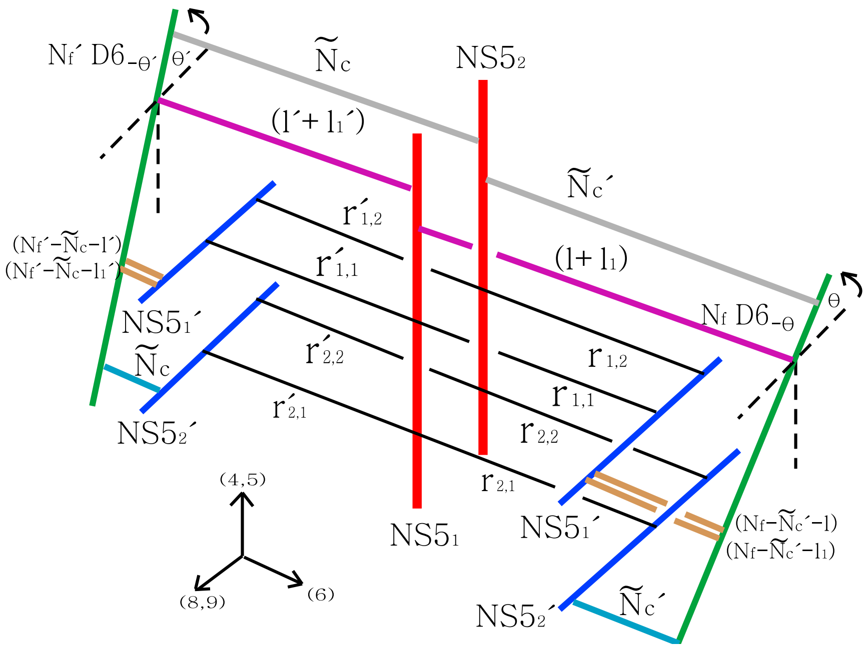

Figure 2: The $\mathcal{N}=1$ supersymmetric magnetic brane configuration corresponding to Figure 1 with a splitting and a reconnection between D4-branes when the gravitational potential of the NS5-branes is ignored. The $NS5_1$-brane is located at $w=0$. The N_f flavor D4-branes connecting between $D6_{-\theta}$-branes and $NS5_1'$-brane are splitting into $(N_f-\widetilde{N}_c'-l_1)$-, l_1-, and $\widetilde{N}_c'$-D4-branes while the other N_f flavor D4-branes connecting between them are splitting into $\widetilde{N}_c'$-, $(N_f-\widetilde{N}_c'-l)$-, and l- D4-branes. Also other $2N_f'$ flavor D4-branes can split similarly. The v and w coordinates of these D4-branes are increasing and decreasing, in that order. Note that the numbers of dual colors connecting i-th $NS5_i'$-brane and j-th $NS5_j$-brane $r_{i,j}$ and $r_{i,j}'$ have the following relations $\sum_{i,j=1}^{2} r_{i,j}=\widetilde{N}_c'-l-l_1$ and $\sum_{i,j=1}^{2} r_{i,j}'=\widetilde{N}_c-l'-l_1'$.

By multiplying f to the second equation of (2.5) from the left, one obtains $fx_1\widetilde{f}=-f\widetilde{f}x_2$. Using the first equation of (2.5) one gets $(-x_2 f)\widetilde{f}=-f\widetilde{f}x_2$ and this leads to $x_2 f\widetilde{f}=f\widetilde{f}x_2$. Then one simplifies the second equation of (2.4) as

$$f\widetilde{f}(x_2\widetilde{q}'M_0+\widetilde{q}'M_1)=0 \rightarrow \widetilde{q}'M_1=-x_2\widetilde{q}'M_0$$

by moving x_2 to the right. Similarly, by multiplying $\widetilde{f}$ to the first equation of (2.5) from the left, one obtains $\widetilde{f}fx_1=-\widetilde{f}x_2 f$. Using the second equation of (2.5) one gets $\widetilde{f}fx_1=(x_1\widetilde{f})f$ and this leads to $x_1\widetilde{f}f=\widetilde{f}fx_1$. Then one simplifies the fifth equation of (2.4) as

$$\widetilde{f}f(x_1\widetilde{q}M_0'+\widetilde{q}M_1')=0 \rightarrow \widetilde{q}M_1'=-x_1\widetilde{q}M_0'$$

by moving x_1 to the right.

Then the remaining F-term equations can be summarized as

$$
\begin{aligned}
& q' f \widetilde{f} x_2 \widetilde{q}' - m + \alpha M_0 = 0, \qquad q \widetilde{f} f x_1 \widetilde{q} - m' + \alpha' M_0' = 0, \qquad q' f \widetilde{f} \widetilde{q}' + \lambda_1 = 0, \\
& q \widetilde{f} f \widetilde{q} + \lambda_2 = 0, \qquad s_1 x_1^2 + \widetilde{f} f (1 + \widetilde{q} M_0' q) = 0, \qquad s_2 x_2^2 + f \widetilde{f} (1 + \widetilde{q}' M_0 q') = 0
\end{aligned} \tag{2.6}
$$

where we used the identities for x_1 and x_2 with $f, \widetilde{f}$ we have discussed.

The theory has many nonsupersymmetric meta-stable ground states and when we rescale the meson fields as $M_0 = h\Lambda\Phi_0$ and $M_0' = h'\Lambda'\Phi_0'$, then the Kahler potential for Φ_0 and Φ_0' is canonical and the magnetic quarks are canonical near the origin of field space [1]. Then the magnetic superpotential (2.3) can be rewritten as

$$
\begin{aligned}
W_{mag} &= \left[h\Phi_0 q' x_2 f \widetilde{f} \widetilde{q}' + \frac{h^2 \mu_\phi}{2} \operatorname{tr} \Phi_0^2 - h\mu^2 \operatorname{tr} \Phi_0 \right] + \left[h'\Phi_0' q x_1 \widetilde{f} f \widetilde{q} + \frac{h'^2 \mu_\phi'}{2} \operatorname{tr} {\Phi_0'}^2 - h' \mu'^2 \operatorname{tr} \Phi_0' \right] \\
&+ \left[\frac{s_1}{3} x_1^3 + \frac{s_2}{3} x_2^3 + x_1 \widetilde{f} f + \widetilde{f} x_2 f + h\Phi_1 (q' f \widetilde{f} \widetilde{q}' + \lambda_1) + h' \Phi_1' (q \widetilde{f} f \widetilde{q} + \lambda_2) \right]
\end{aligned} \tag{2.7}
$$

where $\mu^2 = m\Lambda, \mu'^2 = m'\Lambda'$ and $\mu_\phi = \alpha\Lambda^2, \mu_\phi' = \alpha'\Lambda'^2$.

Now one splits the $(N_f - \widetilde{N}_c' - l) \times (N_f - \widetilde{N}_c' - l)$ block at the lower right corner of $h\Phi_0$ and $q' f \widetilde{f} x_2 \widetilde{q}'$ of supersymmetric solutions into blocks of size n and $(N_f - \widetilde{N}_c' - l - n)$ and one decomposes the $(N_f' - \widetilde{N}_c - l') \times (N_f' - \widetilde{N}_c - l')$ block at the lower right corner of $h'\Phi_0'$ and $q x_1 \widetilde{f} f \widetilde{q}$ of supersymmetric solutions into blocks of size n' and $(N_f' - \widetilde{N}_c - l' - n')$ as follows [3]:

$$
\begin{aligned}
h\Phi_0 &= \begin{pmatrix} \frac{\lambda_1}{\alpha} X_{\widetilde{N}_c'} & 0 & 0 & 0 \\ 0 & 0_l & 0 & 0 \\ 0 & 0 & h\Phi_n & 0 \\ 0 & 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - \widetilde{N}_c' - l - n} \end{pmatrix}, \\
h'\Phi_0' &= \begin{pmatrix} \frac{\lambda_2}{\alpha'} Y_{\widetilde{N}_c} & 0 & 0 & 0 \\ 0 & 0_{l'} & 0 & 0 \\ 0 & 0 & h'\Phi_{n'}' & 0 \\ 0 & 0 & 0 & \frac{\mu'^2}{\mu_\phi'} \mathbf{1}_{N_f' - \widetilde{N}_c - l' - n'} \end{pmatrix}, \\
q' f \widetilde{f} x_2 \widetilde{q}' &= \begin{pmatrix} \mu^2 \mathbf{1}_{\widetilde{N}_c'} - \lambda_1 X_{\widetilde{N}_c'} & 0 & 0 & 0 \\ 0 & \mu^2 \mathbf{1}_l & 0 & 0 \\ 0 & 0 & \varphi' g \widetilde{g} y_2 \widetilde{\varphi}' & 0 \\ 0 & 0 & 0 & 0_{N_f - \widetilde{N}_c' - l - n} \end{pmatrix}, \\
q x_1 \widetilde{f} f \widetilde{q} &= \begin{pmatrix} \mu'^2 \mathbf{1}_{\widetilde{N}_c} - \lambda_2 Y_{\widetilde{N}_c} & 0 & 0 & 0 \\ 0 & \mu'^2 \mathbf{1}_{l'} & 0 & 0 \\ 0 & 0 & \varphi y_1 \widetilde{g} g \widetilde{\varphi} & 0 \\ 0 & 0 & 0 & 0_{N_f' - \widetilde{N}_c - l' - n'} \end{pmatrix},
\end{aligned} \tag{2.8}
$$

with the expectation values of two adjoint fields x_2 and x_1 by $X_{\widetilde{N}'_c} = \text{diag}(a_1, a_2, \cdots, a_{\widetilde{N}'_c})$ and $Y_{\widetilde{N}_c} = \text{diag}(b_1, b_2, \cdots, b_{\widetilde{N}_c})$ which are traceless. We used the first four equations of (2.6) in order to obtain these expectation values. Here φ' and $\widetilde{\varphi}'$ are $n \times (\widetilde{N}'_c - l)$ dimensional matrices and φ and $\widetilde{\varphi}$ are $n' \times (\widetilde{N}_c - l')$ dimensional matrices. In the brane configuration shown in Figure 3, φ' and $\widetilde{\varphi}'$ correspond to fundamental strings connecting the n flavor D4-branes and $(\widetilde{N}'_c - l)$ color D4-branes and φ and $\widetilde{\varphi}$ correspond to fundamental strings connecting the n' flavor D4-branes and $(\widetilde{N}_c - l')$ color D4-branes. The Φ_n and $\varphi' g \widetilde{g} y_2 \widetilde{\varphi}'$ are $n \times n$ matrices while $\Phi'_{n'}$ and $\varphi y_1 \widetilde{g} g \widetilde{\varphi}$ are $n' \times n'$ matrices.

The supersymmetric ground state, in Figure 2, corresponds to $h\Phi_n = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n, \varphi' g y_2 = 0 = y_2 \widetilde{g} \widetilde{\varphi}'$ and $h' \Phi'_n = \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{n'}, \varphi \widetilde{g} y_1 = 0 = y_1 g \widetilde{\varphi}$. Let us make the two NS5'-branes and two NS5-branes be separated along v- and w-directions respectively. Let us put the $NS5'_1$-brane at $v = 0$ and the $NS5_1$-brane at $w = 0$. Then the dual color numbers for each factor are distributed as $r_{i,j}$ and $r'_{i,j}$ connecting between the $NS5'_i$-brane and the $NS5_j$-brane. Then we have $\sum_{i,j=1}^2 r_{i,j} = \widetilde{N}'_c$ and $\sum_{i,j=1}^2 r'_{i,j} = \widetilde{N}_c$.

The l of the N_f-flavor D4-branes are reconnected with l-color D4-branes and the resulting l D4-branes stretch from the $D6_{-\theta}$-branes to the $NS5_1$-brane directly and the intersection point between the l D4-branes and the $D6_{-\theta}$-branes is given by $(v, w) = (+v_{D6_{-\theta}}, 0)$. This corresponds to exactly the l's eigenvalues from zeros of $h\Phi_0$ in (2.8). Now the remaining $(N_f - \widetilde{N}'_c - l)$-flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5'_1$-brane correspond to the eigenvalues of $h\Phi_0$ in (2.8), i.e., $\frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - \widetilde{N}'_c - l}$. The intersection point between the $(N_f - \widetilde{N}'_c - l)$ D4-branes and the $NS5'_1$-branes is given by $(v, w) = (0, +v_{D6_{-\theta}} \cot\theta)$ from trigonometric geometry. Finally, the remnant $\widetilde{N}'_c$-flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5'_2$-brane correspond to the eigenvalues $\frac{\lambda_1}{\alpha} X_{\widetilde{N}'_c}$ in (2.8) providing the positive w coordinates that are $w = +(v_{D6_{-\theta}} - v_{NS5'_2}) \cot\theta$ for these flavor D4-branes.

Similarly, the l' of the N'_f-flavor D4-branes are reconnected with l'-color D4-branes and the resulting l' D4-branes stretch from the $D6_{-\theta'}$-branes to the $NS5_1$-brane directly and the intersection point between the l' D4-branes and the $D6_{-\theta'}$-branes is given by $(v, w) = (+v_{D6_{-\theta'}}, 0)$. This corresponds to exactly the l''s eigenvalues from zeros of $h'\Phi'_0$ in (2.8). Now the remaining $(N'_f - \widetilde{N}_c - l')$-flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5'_1$-brane correspond to the eigenvalues of $h'\Phi'_0$ in (2.8), i.e., $\frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{N'_f - \widetilde{N}_c - l'}$. The intersection point between the $(N'_f - \widetilde{N}_c - l')$ D4-branes and the $NS5'_1$-branes is given by $(v, w) = (0, +v_{D6_{-\theta'}} \cot\theta')$ from trigonometric geometry. Finally, the remnant $\widetilde{N}_c$-flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5'_2$-brane correspond to the eigenvalues $\frac{\lambda_2}{\alpha'} Y_{\widetilde{N}_c}$ in (2.8) providing the positive w coordinates that are $w = +(v_{D6_{-\theta'}} - v_{NS5'_2}) \cot\theta$ for these flavor D4-branes.

Furthermore, one gets the following expectation values by using (2.5)

$$h\Phi_1 = \left(\begin{array}{ccc} -\frac{\lambda_1}{\alpha} X^2_{\widetilde{N}'_c} & 0 & 0 \\ 0 & 0_{l_1} & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - \widetilde{N}'_c - l_1} \end{array} \right), \qquad h'\Phi'_1 = \left(\begin{array}{ccc} -\frac{\lambda_2}{\alpha'} Y^2_{\widetilde{N}_c} & 0 & 0 \\ 0 & 0_{l'_1} & 0 \\ 0 & 0 & \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{N'_f - \widetilde{N}_c - l'_1} \end{array} \right). \quad (2.9)$$

Finally the other two last equations of (2.6) and other equations of (2.5) provide the expectation values for f and $\widetilde{f}$. Note that the superpotential (2.7) has only the linear term in Φ_1 and Φ'_1, contrary to the M_0 and M'_0. In Figure 2 or 3, the l_1 of the N_f-flavor D4-branes are reconnected with l_1-color D4-branes and the resulting l_1 D4-branes stretch from the $D6_{-\theta}$-branes to the $NS5_1$-brane directly and the intersection point between the l_1 D4-branes and the $D6_{-\theta}$-branes is given by $(v, w) = (+v_{D6_{-\theta}}, 0)$. This corresponds to exactly the l_1's eigenvalues from zeros of $h\Phi_1$ in (2.9). Now the remaining $(N_f - \widetilde{N}'_c - l_1)$-flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5'_1$-brane correspond to the eigenvalues of $h\Phi_1$ in (2.9), i.e., $\frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - \widetilde{N}'_c - l_1}$. The intersection point between the $(N_f - \widetilde{N}'_c - l_1)$ D4-branes and the $NS5'_1$-branes is given by $(v, w) = (0, +v_{D6_{-\theta}} \cot\theta)$ from trigonometric geometry [4]. Finally, the remnant $\widetilde{N}'_c$-flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5_2$-brane correspond to the eigenvalues $-\frac{\lambda_1}{\alpha} X^2_{\widetilde{N}'_c}$ in (2.9) providing the negative w coordinates for these flavor D4-branes, that is, the distance between the $NS5_1$-brane and the $NS5_2$-brane.

Similarly, the l'_1 of the N'_f-flavor D4-branes are reconnected with l'_1-color D4-branes and the resulting l'_1 D4-branes stretch from the $D6_{-\theta'}$-branes to the $NS5_1$-brane directly and the intersection point between the l'_1 D4-branes and the $D6_{-\theta'}$-branes is given by $(v, w) = (+v_{D6_{-\theta'}}, 0)$. This corresponds to exactly the l'_1's eigenvalues from zeros of $h'\Phi'_1$ in (2.9). Now the remaining $(N'_f - \widetilde{N}_c - l'_1)$-flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5'_1$-brane correspond to the eigenvalues of $h'\Phi'_1$ in (2.9), i.e., $\frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{N'_f - \widetilde{N}_c - l'_1}$. The intersection point between the $(N'_f - \widetilde{N}_c - l'_1)$ D4-branes and the $NS5'_1$-branes is given by $(v, w) = (0, +v_{D6_{-\theta'}} \cot\theta')$ from trigonometric geometry. Finally, the remnant $\widetilde{N}_c$-flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5_2$-brane correspond to the eigenvalues $-\frac{\lambda_2}{\alpha'} Y^2_{\widetilde{N}_c}$ in (2.9) providing the negative w coordinates for these flavor D4-branes, that is, the distance between the $NS5_1$-brane and the $NS5_2$-brane.

Now the full one loop potential containing $\Phi_n, \Phi'_{n'}, \Phi^\dagger_n$ or $\Phi'^\dagger_{n'}$ with $\mu_\phi << \mu << \Lambda_m$ and $\mu'_\phi << \mu' << \Lambda'_m$, by combining the superpotential and the vacuum expectation values for

the fields, takes the form

$$
\begin{aligned}
V &= |h\Phi_n \varphi' y_2 g \widetilde{g}|^2 + |h g \widetilde{g} y_2 \widetilde{\varphi}' \Phi_n|^2 + |h \varphi' g \widetilde{g} y_2 \widetilde{\varphi}' - h\mu^2 \mathbf{1}_n + h^2 \mu_\phi \Phi_n|^2 + b|h^2\mu|^2 \,\mathrm{tr}\, \Phi_n^\dagger \Phi_n \\
&+ |h' \Phi'_{n'} \varphi y_1 \widetilde{g} g|^2 + |h' \widetilde{g} g y_1 \widetilde{\varphi} \Phi'_{n'}|^2 + |h' \varphi y_1 \widetilde{g} g \widetilde{\varphi} - h' {\mu'}^2 \mathbf{1}_{n'} + {h'}^2 \mu'_\phi \Phi'_{n'}|^2 + b' |{h'}^2 \mu'|^2 \,\mathrm{tr}\, {\Phi'}^\dagger_{n'} \Phi'_{n'} \\
&+ |y_1 \widetilde{g} + \widetilde{g} y_2 + h \widetilde{g} \widetilde{\varphi}' \Phi_n \varphi' y_2 + h' \widetilde{\varphi} \Phi'_n \varphi y_1 \widetilde{g}|^2 + |g y_1 + y_2 g + h \widetilde{\varphi}' \Phi_n \varphi' y_2 g + h' g \widetilde{\varphi} \Phi'_n \varphi y_1|^2 \\
&+ |s_2 y_2^2 + g \widetilde{g} + h g \widetilde{g} \widetilde{\varphi}' \Phi_n \varphi'|^2 + |s_1 y_1^2 + \widetilde{g} g + h' \widetilde{g} g \widetilde{\varphi} \Phi'_n \varphi|^2 + \cdots
\end{aligned}
$$

where $b = \frac{(\ln 4 - 1)}{8\pi^2} \widetilde{N}_c$ and $b' = \frac{(\ln 4-1)}{8\pi^2} \widetilde{N}'_c$ [1] and we did not write down the irrelevant terms which do not depend on Φ_n and $\Phi'_{n'}$ and their conjugate fields explicitly. Differentiating this potential with respect to $\Phi_n^\dagger$ and ${\Phi'}^\dagger_{n'}$ and putting $\varphi' g = 0 = \widetilde{g} \widetilde{\varphi}'$ and $\varphi \widetilde{g} = 0 = g \widetilde{\varphi}$ [3], one obtains

$$
\begin{aligned}
h\Phi_n &\simeq \frac{\mu_\phi}{b} \mathbf{1}_n \qquad \text{or} \qquad M_n \simeq \frac{\alpha \Lambda^3}{\widetilde{N}_c} \mathbf{1}_n, \\
h' \Phi'_{n'} &\simeq \frac{\mu'_\phi}{b'} \mathbf{1}_{n'} \qquad \text{or} \qquad M'_{n'} \simeq \frac{\alpha' {\Lambda'}^3}{\widetilde{N}'_c} \mathbf{1}_{n'}
\end{aligned}
$$

corresponding to the w coordinates of n curved flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5'_1$-brane and the w coordinates of n' curved flavor D4-branes between the $D6_{-\theta'}$-branes and the other $NS5'_1$-brane respectively, in Figure 3. Since $\frac{\mu_\phi}{b} << \frac{\mu^2}{\mu_\phi}$ and $\frac{\mu'_\phi}{b'} << \frac{{\mu'}^2}{\mu'_\phi}$, the n- and n'- curved D4-branes are nearer to $w = 0$ at which the $NS5_1$-brane is located.

One can also consider the fluctuations on the other meson fields Φ_1 and Φ'_1 (2.9) in addition to the above fluctuations. However, the superpotential (2.7) does not contain the quadratic terms for these meson fields, the stable points arise as zero values of w corresponding to supersymmetric vacua. That is, as n_1 or n'_1 D4-branes approach to the $NS5_1$-brane, they reconnect with the same number of color D4-branes respectively and those each combined D4-branes will be connected between $D6_{-\theta}$-branes($D6_{-\theta'}$-branes) and the $NS5_1$-brane directly.

Therefore, the meta-stable states, for cubic superpotential of the adjoint fields with rotation angle $\theta(\theta')$ of $D6_{-\theta}(D6_{-\theta'})$-branes, are classified by the number of various D4-branes $(r_{i,j}, r'_{i,j}, l, l', l_1, l'_1, n, n')$, the position of $D6_{-\theta}$-branes $v_{D6_{-\theta}}$, the position of $D6_{-\theta'}$-branes $v_{D6_{-\theta'}}$, the positions of $NS5'_j$-branes $v_{NS5'_j}$ and the positions of $NS5_j$-branes w_{NS5_j}.

2.3 Higher order superpotential for adjoints

When we consider higher order superpotential whose degree is greater than three(i.e., the number of NS5-branes or NS5'-branes $k > 2$), there exist more meson fields. In general, it is not known how the deformations for the meson fields $Q X_1^j \widetilde{Q}$ or $Q' X_2^j \widetilde{Q}'$ where $j \geq 2$ arise geometrically in the type IIA brane configuration. Therefore, it is not clear how to add these

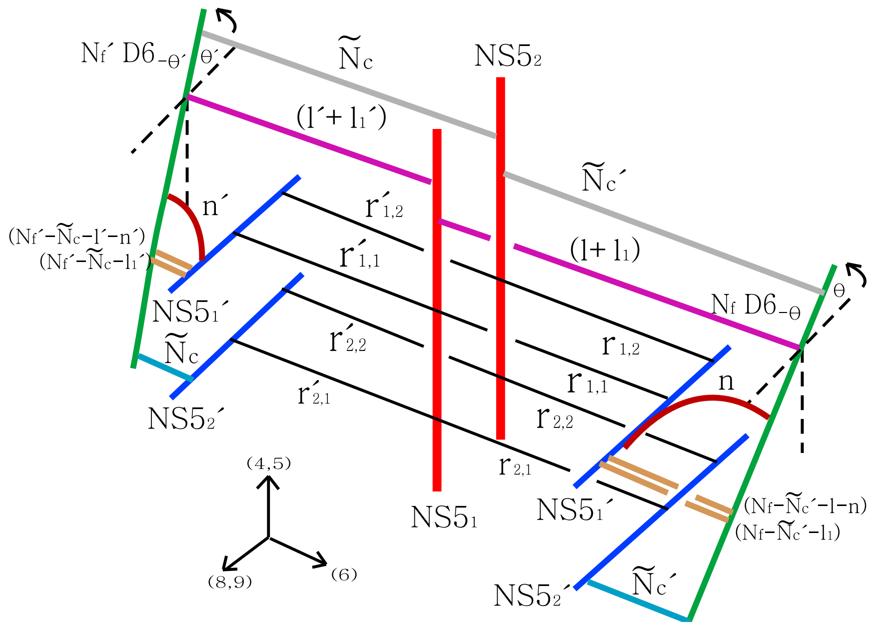

Figure 3: The nonsupersymmetric meta-stable magnetic brane configuration corresponding to Figure 1 with a misalignment between D4-branes when the gravitational potential of the NS5-brane is considered. The $(N_f - \widetilde{N}_c' - l)$ flavor D4-branes in Figure 2 connecting between $D6_{-\theta}$-branes and $NS5_1'$-brane are further splitting into $(N_f - \widetilde{N}_c' - l - n)$- and n-curved D4-branes while the $(N_f' - \widetilde{N}_c - l')$ flavor D4-branes in Figure 2 connecting between $D6_{-\theta'}$-branes and $NS5_1'$-brane are further splitting into $(N_f' - \widetilde{N}_c - l' - n')$- and n'-curved D4-branes.

deformations in an electric theory or a magnetic theory. Recall that the perturbations by $QX_1\widetilde{Q}$ and $Q'X_2\widetilde{Q}'$ when $j = 1$ in the superpotential were realized by the relative rotations of $D6_{-\theta}(D6_{-\theta'})$-branes in (w, v)-plane.

2.4 $SU(N_c)$ with N_f-fund., symmetric and conjugate symmetric tensors, and adjoint.

When we add an orientifold 6-plane to the above brane configuration, then we obtain this theory. Due to the orientifold 6-plane, the number of gauge group, fundamentals, and adjoint fields are reduced by half. Moreover, the bifundamentals become symmetric and conjugate symmetric tensors. The type IIA brane configuration for this theory is already known in [23]. The gauge theory analysis was found in [30] where the power of adjoint field in the superpotential appears in $k + 1$ and they found that k should be odd. For the simplest nontrivial case, one sets $k = 3$. The number of NS5-branes or NS5'-branes is equal to three also. Then there exist three meson fields: $M_0 \equiv Q\widetilde{Q}, M_1 \equiv QX\widetilde{Q}$, and $M_2 \equiv QXX\widetilde{Q}$. Since we do not know how the deformation for M_2 arises geometrically in the type IIA brane

configuration, it is not clear how to add the deformation corresponding to M_2 in an electric theory or a magnetic theory.

3 $SO(2N_c) \times Sp(N_c')$ with $2N_f$-vectors, $2N_f'$-fund., two adjoints, and bifund.

Let us add the O4-plane to the brane configuration of previous section.

3.1 Electric theory

The type IIA supersymmetric electric brane configuration corresponding to $\mathcal{N}=1$ $SO(2N_c)\times Sp(N_c')$ gauge theory [31] with $2N_f$-vector Q, $2N_f'$-fundamental fields Q', bifundamentals F and two adjoint fields X_1, X_2 can be described as two middle NS5-branes, two left NS5'-branes, two right NS5'-branes, $2N_c$- and $2N_c'$-D4-branes, $2N_f$- and $2N_f'$-D6-branes and an O4-plane for the cubic superpotential of the adjoints. The X_1 is in the representation $(\mathbf{N_c(2N_c-1)}, \mathbf{1})$ while the X_2 is in the representation $(\mathbf{1}, \mathbf{N_c'(2N_c'+1)})$, under the gauge group. The F is in the representation $(\mathbf{2N_c}, \mathbf{2N_c'})$ under the gauge group. The quarks Q is in the representation $(\mathbf{2N_c}, \mathbf{1})$ and the quarks Q' is in the representation $(\mathbf{1}, \mathbf{2N_c'})$, under the gauge group.

The mass terms for each quarks can be added by displacing each D6-branes along v direction leading to their coordinates $v = \pm v_{D6_{-\theta}}(\pm v_{D6_{-\theta'}})$ respectively while the quartic terms for each quarks can be added also by rotating each D6-branes by an angle $-\theta(-\theta')$ in (w,v)-plane respectively. Then, the general superpotential by adding the above deformations is given by

$$\begin{aligned} W_{elec} &= \left[\frac{s_1}{3}\,\mathrm{tr}\,X_1^3 + \frac{s_2}{3}\,\mathrm{tr}\,X_2^3 + \mathrm{tr}\,X_1F^2 + \mathrm{tr}\,X_2F^2 + \lambda_1 QX_1Q + \lambda_2 Q'X_2Q'\right] \\ &+ \frac{\alpha}{2}\,\mathrm{tr}(QQ)^2 - m\,\mathrm{tr}\,QQ + \frac{\alpha'}{2}\,\mathrm{tr}(Q'Q')^2 - m'\,\mathrm{tr}\,Q'Q'. \end{aligned} \tag{3.1}$$

The parameters are the same as before and the rotation angles ω_L and ω_R of two left and right NS5'-branes with respect to the middle NS5-branes are given by $\omega_L = \omega_R = \frac{\pi}{2}$. Although the relative displacement of two color D4-branes can be added in the superpotential, we focus on the particular zero limit of bifundamental mass. There are the perturbations by QX_1Q and $Q'X_2Q'$ in the superpotential which will arise as the mesons in the magnetic theory. The mass matrix m is symmetric and the mass matrix m' is antisymmetric.

Then the $\mathcal{N}=1$ supersymmetric electric brane configuration for the superpotential (3.1) in type IIA string theory is given as follows:

- Two middle NS5-branes in (012345) directions

- Two left NS5'-branes in (012389) directions
- Two right NS5'-branes in (012389) directions
- $2N_f$ $D6_{-\theta}$-branes in (01237) directions and two other directions in (v, w)-plane
- $2N'_f$ $D6_{-\theta'}$-branes in (01237) directions and two other directions in (v, w)-plane
- $2N_c$- and $2N'_c$-color D4-branes in (01236) directions
- $O4^{\pm}$-planes in (01236) directions

The corresponding brane configuration can be obtained from the previous section by considering the correct mirrors based on the O4-plane action.

3.2 Magnetic theory

It is straightforward to compute the dual color numbers by considering the D4-brane charge of an orientifold 4-plane and realizing that the number of D6-branes are doubled.

The left NS5'-branes start out with linking number $l_e = -\frac{4N'_f}{2} + 2N_c - 2$ and after duality these left NS5'-branes end up with linking number $l_m = \frac{4N'_f}{2} - 2\widetilde{N}'_c + 4N_f - 2$. We consider only the particular brane motion where N_f $D6_{-\theta}$-branes meet the middle NS5-branes with no angles(and their mirrors). That is, the $D6_{-\theta}$-branes become $D6_{-\frac{\pi}{2}}$-branes when they meet with the middle NS5-branes instantaneously and then after that they come back to the original $D6_{-\theta}$-branes. Therefore, in this dual process, there is no creation of D4-branes. That is the reason for the $4N_f$ factor in the l_m, not $8N_f$. Then the dual color number $2\widetilde{N}'_c$ is given by $2\widetilde{N}'_c = 4N_f + 4N'_f - 2N_c$.

The right NS5'-branes start out with linking number $l_e = \frac{4N_f}{2} - 2N'_c - 2$ and after duality these right NS5'-branes end up with linking number $l_m = -\frac{4N_f}{2} + 2\widetilde{N}_c - 4N'_f - 2$. We consider only the particular brane motion where the $D6_{-\theta'}$-branes become $D6_{-\frac{\pi}{2}}$-branes when they meet with the middle NS5-branes instantaneously and after that they come back to the original $D6_{-\theta'}$-branes(and their mirrors). Therefore, in this dual process, there is no creation of D4-branes. That is the reason for the $4N'_f$ factor in the l_m, not $8N'_f$. Then it turns out that the dual color number $2\widetilde{N}_c$ is given by $2\widetilde{N}_c = 4N'_f + 4N_f - 2N'_c$. Finally, one has the following dual color numbers

$$2\widetilde{N}_c = 4N'_f + 4N_f - 2N'_c, \qquad 2\widetilde{N}'_c = 4N_f + 4N'_f - 2N_c.$$

The low energy theory on the two color D4-branes has $SO(2\widetilde{N}_c) \times Sp(\widetilde{N}'_c)$ gauge group and $2N_f$-vector dual quarks q', $2N'_f$-fundamental dual quarks q, bifundamental f and various gauge singlets. The f is in the representation $(\mathbf{2\widetilde{N}_c}, \mathbf{2\widetilde{N}'_c})$ under the dual gauge group. The $2N'_f$ fields q are in the representation $(\mathbf{2\widetilde{N}_c}, \mathbf{1})$ and similarly, the $2N_f$ fields q' are in the representation $(\mathbf{1}, \mathbf{2\widetilde{N}'_c})$, under the gauge group. In particular, a magnetic meson field $M_0 \equiv QQ$ is $2N_f \times 2N_f$

matrix and comes from 4-4 strings of $2N_f$ flavor D4-branes created when $2N_f$ $D6_{-\theta}$-branes meet the one of the right NS5'-branes while a magnetic meson field $M_0' \equiv Q'Q'$ is $2N_f' \times 2N_f'$ matrix and comes from 4-4 strings of $2N_f'$ flavor D4-branes created when $2N_f'$ $D6_{-\theta'}$-branes meet the one of the left NS5'-branes. The adjoint fields x_1, x_2 correspond to the motion of the NS5'-branes and the NS5-branes in (v, w)-plane.

Then the most general magnetic superpotential, when we consider the case where $N_f(N_f')$ $D6_{-\theta}$-branes($D6_{-\theta'}$-branes) meet the middle NS5-branes *with angles*(and their mirrors), is given by

$$
\begin{aligned}
W_{dual} &= \left[\frac{s_1}{3}x_1^3 + \frac{s_2}{3}x_2^3 + x_1 f^2 + x_2 f^2 + \lambda_1 M_1 + \lambda_2 M_1'\right] \\
&+ \left(\frac{\alpha}{2}\,\mathrm{tr}\, M_0^2 - m M_0\right) + \left(\frac{\alpha'}{2}\,\mathrm{tr}\, {M_0'}^2 - m' M_0'\right) \\
&+ \left[M_0 q' x_2 f^2 q' + M_0' q x_1 f^2 q + M_1 q' f^2 q' + M_1' q f^2 q\right] \\
&+ \left[M_2 q' x_2 q' + M_3 q' q' + M_2' q x_1 q + M_3' q q + P_1 q x_1 f q' + P_2 q f q' + \widetilde{P}_2 q' f q\right] \qquad (3.2)
\end{aligned}
$$

where the mesons are given by

$$
\begin{aligned}
M_0 &\equiv QQ, \quad M_0' \equiv Q'Q', \quad M_1 \equiv QX_1Q, \quad M_1' \equiv Q'X_2Q', \\
M_2 &\equiv QF^2Q, \quad M_3 \equiv QF^2X_1Q, \quad M_2' \equiv Q'F^2Q', \quad M_3' \equiv Q'F^2X_2Q', \\
P_1 &\equiv QFQ', \quad P_2 \equiv QX_1FQ', \quad \widetilde{P}_2 \equiv Q'X_2FQ.
\end{aligned}
$$

The first two lines of (3.2) are dual expressions for the electric superpotential (3.1) and the corresponding meson fields M_0, M_0', M_1 and M_1' are replaced and the third and fourth lines of (3.2) are the analogs of the cubic term superpotential between the meson and dual quarks in Seiberg duality. Compared with the theory [24, 10] without two adjoints fields, there exist the extra meson fields coming from the adjoint fields X_1 and X_2:$M_1, M_1', M_3, M_3', P_2$ and $\widetilde{P}_2$.

Let us find out the relevant magnetic superpotential for the meta-stable brane configuration.

When the N_f $D6_{-\theta}$-branes meet the middle NS5-branes(and their mirrors), no creation of D4-branes implies that there is no M_2- or M_3-term in the above superpotential (3.2). The mesons M_2 and M_3 originate from $SO(2N_c)$ chiral mesons QQ when one dualizes the $SO(2N_c)$ gauge group first by moving the middle NS5-branes to the left of the left NS5'-branes [32]. That is, the fluctuations of strings stretching between the $2N_f$ "flavor" D4-branes correspond to these meson fields(and their mirrors). After the additional dual procedures, the cubic terms in the superpotential arise as M_2-dependent and M_3-dependent terms where M_2 has extra F^2 fields and M_3 has extra F^2X_1 fields, besides QQ, due to the further dualization. The M_2-term in the superpotential has an extra x_2 factor besides $q'q'$.

Similarly, when the N_f' $D6_{-\theta'}$-branes meet the middle NS5-branes with no angles(and their mirrors), there is no M_2'- or M_3'-term in the above superpotential (3.2). These meson fields M_2' and M_3' originate from $Sp(N_c')$ chiral mesons $Q'Q'$ when one dualizes the $Sp(N_c')$ gauge group first by moving the middle NS5-branes to the right of the right NS5'-branes. The strings stretching between the $2N_f'$ "flavor" D4-branes provide these mesons(and their mirrors). After the additional dual procedures, the cubic terms in the superpotential arise as M_2'-term and M_3'-term where M_2' has extra F^2 fields and M_3' has extra F^2X_2 fields, besides $Q'Q'$, due to the further dualization. The M_2'-term in the superpotential has an extra x_1 factor besides qq.

Furthermore, when the N_f $D6_{-\theta}$-branes, the N_f' $D6_{-\theta'}$-branes and the middle NS5-branes meet each other with no angles(and their mirrors), no P_1- and P_2- or $\widetilde{P}_2$-dependent terms occur in the superpotential (3.2). These mesons originate from $Sp(N_c')$ chiral mesons FQ' when one dualizes the $Sp(N_c')$ first by moving the middle NS5-branes to the right of the right NS5'-branes. The strings stretching between the $2N_f'$ flavor D4-branes and $2N_c$ color D4-branes give rise to these $2N_f'$ $SO(2N_c)$ vectors(and their mirrors). After the additional dual procedures, these cubic terms arise as these meson terms where there exist extra qx_1, q and q in the interactions of P_1, P_2 and $\widetilde{P}_2$ in the superpotential and the mesons have extra Q, QX_1, QX_2 fields respectively, due to the further dualization.

Then the reduced magnetic superpotential in our case by taking the first three lines of (3.2) is given by

$$\begin{aligned} W_{dual} &= \left[\frac{s_1}{3}x_1^3 + \frac{s_2}{3}x_2^3 + x_1 f^2 + f x_2 f + M_1(q' f^2 q' + \lambda_1) + M_1'(q f^2 q + \lambda_2)\right] \\ &+ \left[M_0 q' x_2 f^2 q' + \frac{\alpha}{2}\operatorname{tr} M_0^2 - m M_0\right] + \left[M_0' q x_1 f^2 q + \frac{\alpha'}{2}\operatorname{tr} M_0'^2 - m' M_0'\right]. \end{aligned} \tag{3.3}$$

Let us describe the meta-stable brane configuration with this magnetic superpotential.

For the supersymmetric vacua, one can compute the F-term equations for this superpotential (3.3) and the F-terms for $M_0, q', M_0', q, f, M_1, M_1', x_1$ and x_2 are given by

$$\begin{aligned} & q'x_2 f^2 q' - m + \alpha M_0 = 0, \qquad x_2 f^2 q' M_0 + f^2 q' M_1 + (M_0 q' x_2 + M_1 q') f^2 = 0, \\ & q x_1 f^2 q - m' + \alpha' M_0' = 0, \qquad x_1 f^2 q M_0' + f^2 q M_1' + (M_0' q x_1 + M_1' q) f^2 = 0, \\ & (x_1 f + f x_2) + f q'(M_1 q' + M_0 q' x_2) + q(M_1' q + M_0' q x_1) f \\ & +(f x_1 + x_2 f) + q'(M_1 q' + M_0 q' x_2) f + f q(M_1' q + M_0' q x_1) = 0, \\ & q' f^2 q' + \lambda_1 = 0, \qquad q f^2 q + \lambda_2 = 0, \\ & s_1 x_1^2 + f^2 + f^2 q M_0' q = 0, \qquad s_2 x_2^2 + f^2 + f^2 q' M_0 q' = 0. \end{aligned} \tag{3.4}$$

The fifth equation of (3.4) is satified if the following equations hold

$$f x_1 = -x_2 f, \qquad x_1 f = -f x_2, \qquad M_1 q' = -M_0 q' x_2, \qquad M_1' q = -M_0' q x_1. \tag{3.5}$$

By multiplying f to the second equation of (3.5), one obtains $f x_1 f = -f^2 x_2$. Using the first equation of (3.5) one gets $(-x_2 f) f = -f^2 x_2$ and this leads to $x_2 f^2 = f^2 x_2$. Then one simplifies the second equation of (3.4) with (3.5) as

$$f^2(x_2 q' M_0 + q' M_1) = 0 \rightarrow q' M_1 = -x_2 q' M_0$$

by moving x_2 to the right. Similarly, by multiplying f to the first equation of (3.5), one obtains $f^2 x_1 = -f x_2 f$. Using the second equation of (3.5) one gets $f^2 x_1 = (x_1 f) f$ and this leads to $x_1 f^2 = f^2 x_1$. Then one simplifies the fourth equation of (3.4) with (3.5) as

$$f^2(x_1 q M_0' + q M_1') = 0 \rightarrow q M_1' = -x_1 q M_0'$$

by moving x_1 to the right.

Then the remaining F-term equations can be summarized as

$$\begin{aligned} & q' f^2 x_2 q' - m + \alpha M_0 = 0, \qquad q f^2 x_1 q - m' + \alpha' M_0' = 0, \qquad q' f^2 q' + \lambda_1 = 0, \\ & q f^2 q + \lambda_2 = 0, \qquad s_1 x_1^2 + f^2(1 + q M_0' q) = 0, \qquad s_2 x_2^2 + f^2(1 + q' M_0 q') = 0 \end{aligned} \tag{3.6}$$

where we used the identities for x_1 and x_2 with f we discussed.

The theory has many nonsupersymmetric meta-stable ground states and when we rescale the meson fields as $M_0 = h\Lambda\Phi_0$ and $M_0' = h'\Lambda'\Phi_0'$, then the Kahler potential for Φ_0 and Φ_0' is canonical and the magnetic quarks are canonical near the origin of field space. Then the magnetic superpotential (3.3) can be rewritten as

$$\begin{aligned} W_{mag} &= \left[h\Phi_0 q' x_2 f^2 q' + \frac{h^2 \mu_\phi}{2} \operatorname{tr} \Phi_0^2 - h\mu^2 \operatorname{tr} \Phi_0 \right] + \left[h'\Phi_0' q x_1 f^2 q + \frac{h'^2 \mu_\phi'}{2} \operatorname{tr} {\Phi_0'}^2 - h' \mu'^2 \operatorname{tr} \Phi_0' \right] \\ &+ \left[\frac{s_1}{3} x_1^3 + \frac{s_2}{3} x_2^3 + x_1 f^2 + f x_2 f + h\Phi_1 (q' f^2 q' + \lambda_1) + h'\Phi_1' (q f^2 q + \lambda_2) \right] \end{aligned} \tag{3.7}$$

where $\mu^2 = m\Lambda, \mu'^2 = m'\Lambda'$ and $\mu_\phi = \alpha\Lambda^2, \mu_\phi' = \alpha'\Lambda'^2$.

Now one splits the $2(N_f - \widetilde{N}_c' - l) \times 2(N_f - \widetilde{N}_c' - l)$ block at the lower right corner of $h\Phi_0$ and $q' f^2 x_2 q'$ into blocks of size $2n$ and $2(N_f - \widetilde{N}_c' - l - n)$ and one decomposes the $2(N_f' - \widetilde{N}_c - l') \times 2(N_f' - \widetilde{N}_c - l')$ block at the lower right corner of $h'\Phi_0'$ and $q x_1 f^2 q$ into blocks

of size $2n'$ and $2(N_f' - \widetilde{N}_c - l' - n')$ as follows:

$$
\begin{aligned}
h\Phi_0 &= \begin{pmatrix} \frac{\lambda_1}{\alpha} X_{2\widetilde{N}_c'} & 0 & 0 & 0 \\ 0 & 0_{2l} & 0 & 0 \\ 0 & 0 & h\Phi_{2n} & 0 \\ 0 & 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - \widetilde{N}_c' - l - n} \otimes \sigma_3 \end{pmatrix}, \\
h'\Phi_0' &= \begin{pmatrix} \frac{\lambda_2}{\alpha'} Y_{2\widetilde{N}_c} & 0 & 0 & 0 \\ 0 & 0_{2l'} & 0 & 0 \\ 0 & 0 & h'\Phi_{2n'}' & 0 \\ 0 & 0 & 0 & \frac{\mu'^2}{\mu_\phi'} \mathbf{1}_{N_f' - \widetilde{N}_c - l' - n'} \otimes i\sigma_2 \end{pmatrix}, \\
q' f^2 x_2 q' &= \begin{pmatrix} \mu^2 \mathbf{1}_{2\widetilde{N}_c'} - \lambda_1 X_{2\widetilde{N}_c'} & 0 & 0 & 0 \\ 0 & \mu^2 \mathbf{1}_{2l} & 0 & 0 \\ 0 & 0 & \varphi' g^2 y_2 \varphi' & 0 \\ 0 & 0 & 0 & 0_{2(N_f - \widetilde{N}_c' - l - n)} \end{pmatrix}, \\
q x_1 f^2 q &= \begin{pmatrix} \mu'^2 \mathbf{1}_{2\widetilde{N}_c} - \lambda_2 Y_{2\widetilde{N}_c} & 0 & 0 & 0 \\ 0 & \mu'^2 \mathbf{1}_{2l'} & 0 & 0 \\ 0 & 0 & \varphi y_1 g^2 \varphi & 0 \\ 0 & 0 & 0 & 0_{2(N_f' - \widetilde{N}_c - l' - n')} \end{pmatrix},
\end{aligned}
\tag{3.8}
$$

with $X_{2\widetilde{N}_c'} = \mathrm{diag}(a_1, a_2, \cdots, a_{\widetilde{N}_c'}) \otimes \sigma_3$ and $Y_{2\widetilde{N}_c} = \mathrm{diag}(b_1, b_2, \cdots, b_{\widetilde{N}_c}) \otimes i\sigma_2$. We used the first four equations of (3.6) in order to obtain these expectation values. Here φ' is $2n \times 2(\widetilde{N}_c' - l)$ dimensional matrices and φ is $2n' \times 2(\widetilde{N}_c - l')$ dimensional matrices. The φ' corresponds to fundamental strings connecting the $2n$ flavor D4-branes and $2(\widetilde{N}_c' - l)$ color D4-branes and φ corresponds to fundamental strings connecting the $2n'$ flavor D4-branes and $2(\widetilde{N}_c - l')$ color D4-branes. The Φ_{2n} and $\varphi' g^2 y_2 \varphi'$ are $2n \times 2n$ matrices while $\Phi_{2n'}'$ and $\varphi y_1 g^2 \varphi$ are $2n' \times 2n'$ matrices.

The supersymmetric ground state corresponds to $h\Phi_{2n} = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n \otimes \sigma_3, \varphi' g y_2 = 0 = y_2 g \varphi'$ and $h'\Phi_{2n'}' = \frac{\mu'^2}{\mu_\phi'} \mathbf{1}_{n'} \otimes i\sigma_2, \varphi g y_1 = 0 = y_1 g \varphi$. The l of the upper N_f-flavor D4-branes are reconnected with l-color D4-branes and the resulting l D4-branes stretch from the upper $D6_{-\theta}$-branes to the $NS5_1$-brane directly and the intersection point between the l D4-branes and the upper $D6_{-\theta}$-branes is given by $(v, w) = (+v_{D6_{-\theta}}, 0)$. The mirrors are located at $(v, w) = (-v_{D6_{-\theta}}, 0)$. This corresponds to exactly the $2l$'s eigenvalues from zeros of $h\Phi_0$ in (3.8). Now the remaining upper $(N_f - \widetilde{N}_c' - l)$-flavor D4-branes between the upper $D6_{-\theta}$-branes and the $NS5_1'$-brane correspond to the positive eigenvalues of $h\Phi_0$ in (3.8), i.e., $\frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - \widetilde{N}_c' - l}$. The intersection point between the upper $(N_f - \widetilde{N}_c' - l)$ D4-branes and the $NS5_1'$-branes is given by $(v, w) = (0, +v_{D6_{-\theta}} \cot\theta)$ from trigonometric geometry. The mirrors are located at $(v, w) = (0, -v_{D6_{-\theta}} \cot\theta)$ corresponding to the negative eigenvalues of $h\Phi_0$ in (3.8), i.e.,

$-\frac{\mu^2}{\mu_\phi}\mathbf{1}_{N_f-\widetilde{N}'_c-l}$. Finally, the remnant $2\widetilde{N}'_c$-flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5'_2$-brane correspond to the eigenvalues $\frac{\lambda_1}{\alpha}X_{2\widetilde{N}'_c}$ in (3.8) providing the w coordinates that are $w = +(v_{D6_{-\theta}} - v_{NS5'_2})\cot\theta$ for these flavor D4-branes.

Similarly, the l' of the upper N'_f-flavor D4-branes are reconnected with l'-color D4-branes and the resulting l' D4-branes stretch from the upper $D6_{-\theta'}$-branes to the $NS5_1$-brane directly and the intersection point between the l' D4-branes and the upper $D6_{-\theta'}$-branes is given by $(v,w) = (+v_{D6_{-\theta'}}, 0)$. The mirrors are located at $(v,w) = (-v_{D6_{-\theta'}}, 0)$. This corresponds to exactly the $2l'$'s eigenvalues from zeros of $h'\Phi'_0$ in (3.8). Now the remaining upper $(N'_f-\widetilde{N}_c-l')$-flavor D4-branes between the upper $D6_{-\theta'}$-branes and the $NS5'_1$-brane correspond to the positive eigenvalues of $h'\Phi'_0$ in (3.8), i.e., $\frac{\mu'^2}{\mu'_\phi}\mathbf{1}_{N'_f-\widetilde{N}_c-l'}$. The intersection point between the $(N'_f - \widetilde{N}_c - l')$ D4-branes and the $NS5'_1$-branes is given by $(v,w) = (0, +v_{D6_{-\theta'}}\cot\theta')$ from trigonometric geometry. The mirrors are located at $(v,w) = (0, -v_{D6_{-\theta'}}\cot\theta')$ corresponding to the negative eigenvalues of $h'\Phi'_0$ in (3.8), i.e., $-\frac{\mu'^2}{\mu'_\phi}\mathbf{1}_{N'_f-\widetilde{N}_c-l'}$. Finally, the remnant $2\widetilde{N}_c$-flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5'_2$-brane correspond to the eigenvalues $\frac{\lambda_2}{\alpha'}Y_{2\widetilde{N}_c}$ in (3.8) providing the w coordinates that are $w = +(v_{D6_{-\theta'}} - v_{NS5'_2})\cot\theta'$ for these flavor D4-branes.

Furthermore, one gets the following expectation values by using (3.5)

$$
\begin{aligned}
h\Phi_1 &= \begin{pmatrix} -\frac{\lambda_1}{\alpha}X^2_{2\widetilde{N}'_c} & 0 & 0 \\ 0 & 0_{2l_1} & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi}\mathbf{1}_{N_f-\widetilde{N}'_c-l_1}\otimes\sigma_3 \end{pmatrix}, \\
h'\Phi'_1 &= \begin{pmatrix} -\frac{\lambda_2}{\alpha'}Y^2_{2\widetilde{N}_c} & 0 & 0 \\ 0 & 0_{2l'_1} & 0 \\ 0 & 0 & \frac{\mu'^2}{\mu'_\phi}\mathbf{1}_{N'_f-\widetilde{N}_c-l'_1}\otimes i\sigma_2 \end{pmatrix}.
\end{aligned}
\tag{3.9}
$$

Finally the other two last equations of (3.6) and other equation of (3.5) provide the expectation value for f. Note that the superpotential (3.7) has only the linear term in Φ_1 and Φ'_1, contrary to the M_0 and M'_0. The l_1 of the upper N_f-flavor D4-branes are reconnected with l_1-color D4-branes and the resulting l_1 D4-branes stretch from the upper $D6_{-\theta}$-branes to the $NS5_1$-brane directly and the intersection point between the l_1 D4-branes and the upper $D6_{-\theta}$-branes is given by $(v,w) = (+v_{D6_{-\theta}}, 0)$. The mirrors are located at $(v,w) = (-v_{D6_{-\theta}}, 0)$. This corresponds to exactly the $2l_1$'s eigenvalues from zeros of $h\Phi_1$ in (3.9). Now the remaining upper $(N_f - \widetilde{N}'_c - l_1)$-flavor D4-branes between the upper $D6_{-\theta}$-branes and the $NS5'_1$-brane correspond to the eigenvalues of $h\Phi_1$ in (3.9), i.e., $\frac{\mu^2}{\mu_\phi}\mathbf{1}_{N_f-\widetilde{N}'_c-l_1}$. The intersection point between the upper $(N_f-\widetilde{N}'_c-l_1)$ D4-branes and the $NS5'_1$-branes is given by $(v,w) = (0, +v_{D6_{-\theta}}\cot\theta)$ from trigonometric geometry. The mirrors are located at $(v,w) = (0, -v_{D6_{-\theta}}\cot\theta)$. Finally,

the remnant $2\widetilde{N}_c'$-flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5_2$-brane correspond to the eigenvalues $-\frac{\lambda_1}{\alpha}X^2_{2\widetilde{N}_c'}$ in (3.9) providing the negative w coordinates for these flavor D4-branes.

Similarly, the l_1' of the upper N_f'-flavor D4-branes are reconnected with l_1'-color D4-branes and the resulting l_1' D4-branes stretch from the upper $D6_{-\theta'}$-branes to the $NS5_1$-brane directly and the intersection point between the l_1' D4-branes and the $D6_{-\theta'}$-branes is given by $(v,w) = (+v_{D6_{-\theta'}}, 0)$. The mirrors are located at $(v,w) = (-v_{D6_{-\theta'}}, 0)$. This corresponds to exactly the $2l_1'$'s eigenvalues from zeros of $h'\Phi_1'$ in (3.9). Now the remaining upper $(N_f' - \widetilde{N}_c - l_1')$-flavor D4-branes between the upper $D6_{-\theta'}$-branes and the $NS5_1'$-brane correspond to the eigenvalues of $h'\Phi_1'$ in (3.9), i.e., $\frac{\mu'^2}{\mu_\phi'}\mathbf{1}_{N_f'-\widetilde{N}_c-l_1'}$. The intersection point between the $(N_f' - \widetilde{N}_c - l_1')$ D4-branes and the $NS5_1'$-branes is given by $(v,w) = (0, +v_{D6_{-\theta'}}\cot\theta')$ from trigonometric geometry and the mirrors are located at $(v,w) = (0, -v_{D6_{-\theta'}}\cot\theta')$. Finally, the remnant $2\widetilde{N}_c$-flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5_2$-brane correspond to the eigenvalues $-\frac{\lambda_2}{\alpha'}Y^2_{2\widetilde{N}_c}$ in (3.9) providing the negative w coordinates for these flavor D4-branes.

Now the full one loop potential containing $\Phi_{2n}, \Phi_{2n'}'$, by combining the superpotential and the vacuum expectation values for the fields, takes the form

$$\begin{aligned} V &= |h\Phi_{2n}\varphi' y_2 g^2 + hg^2 y_2\varphi'\Phi_{2n}|^2 + |h\varphi' g^2 y_2 \varphi' - h\mu^2\mathbf{1}_{2n} + h^2\mu_\phi\Phi_{2n}|^2 + b|h^2\mu|^2 \,\mathrm{tr}\, \Phi_{2n}\Phi_{2n} \\ &+ |h'\Phi_{2n'}'\varphi y_1 g^2 + h'g^2 y_1\varphi\Phi_{2n'}'|^2 + |h'\varphi y_1 g^2\varphi - h'\mu'^2\mathbf{1}_{2n'} + h'^2\mu_\phi'\Phi_{2n'}'|^2 + b'|h'^2\mu'|^2 \,\mathrm{tr}\, \Phi'_{2n'}\Phi_{2n'}' \\ &+ |y_1 g + g y_2 + hg\varphi'\Phi_{2n}\varphi' y_2 + h'\varphi\Phi_{2n'}\varphi y_1 g + g y_1 + y_2 g + h\varphi'\Phi_{2n}\varphi' y_2 g + h' g\varphi\Phi_{2n'}\varphi y_1|^2 \\ &+ |s_2 y_2^2 + g^2 + hg^2\varphi'\Phi_{2n}\varphi'|^2 + |s_1 y_1^2 + g^2 + h'g^2\varphi\Phi_{2n}'\varphi|^2 + \cdots \end{aligned}$$

where $b = \frac{(\ln 4 - 1)}{8\pi^2}\widetilde{N}_c$ and $b' = \frac{(\ln 4-1)}{8\pi^2}\widetilde{N}_c'$ [1, 33] and we did not write down the irrelevant terms which do not depend on Φ_{2n} and $\Phi_{2n'}'$ explicitly. Differentiating this potential with respect to Φ_{2n} and $\Phi'_{2n'}$ and putting $\varphi' g = 0 = g\varphi'$ and $\varphi g = 0 = g\varphi$, one obtains

$$\begin{aligned} h\Phi_{2n} &\simeq \frac{\mu_\phi}{b}\mathbf{1}_n \otimes \sigma_3 \qquad \text{or} \qquad M_{2n} \simeq \frac{\alpha\Lambda^3}{\widetilde{N}_c}\mathbf{1}_n \otimes \sigma_3, \\ h'\Phi_{2n'}' &\simeq \frac{\mu_\phi'}{b'}\mathbf{1}_{n'} \otimes i\sigma_2 \qquad \text{or} \qquad M_{2n'}' \simeq \frac{\alpha'\Lambda'^3}{\widetilde{N}_c'}\mathbf{1}_{n'} \otimes i\sigma_2 \end{aligned}$$

corresponding to the w coordinates of $2n$ curved flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5_1'$-brane and the w coordinates of $2n'$ curved flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5_1'$-brane respectively.

One can also consider the fluctuations on the other meson fields Φ_1 and Φ_1' (3.9) in addition to the above fluctuations. Since the superpotential (3.7) does not contain the quadratic terms for these meson fields, the stable points arise as zero values of w corresponding to

supersymmetric vacua. That is, as $2n_1$ or $2n'_1$ D4-branes approach to the $NS5_1$-brane, they reconnect with the same number of color D4-branes respectively and those each combined D4-branes will be connected between $D6_{-\theta}$-branes($D6_{-\theta'}$-branes) and the $NS5_1$-brane directly.

3.3 Higher order superpotential for adjoints

When we consider higher order superpotential whose degree is greater than three(i.e., the number of NS5-branes or NS5'-branes $k > 2$), there exist more meson fields. In general, it is not known how the deformations for the meson fields QX_1^jQ or $Q'X_2^jQ'$ where $j \geq 2$ arise geometrically in the type IIA brane configuration, as in previous section. Therefore, it is not clear how to add these deformations in an electric theory or a magnetic theory. Recall that the perturbations by QX_1Q and $Q'X_2Q'$ when $j = 1$ in the superpotential were realized by the relative rotations of $D6_{-\theta}(D6_{-\theta'})$-branes in (w, v)-plane.

4 $SU(N_c)\times SU(N'_c)\times SU(N''_c)$ with N_f-, N'_f-, and N''_f-fund., and bifund.

4.1 Electric theory

The type IIA supersymmetric electric brane configuration [15, 22] corresponding to $\mathcal{N} = 1$ $SU(N_c)\times SU(N'_c)\times SU(N''_c)$ gauge theory with N_f-fundamental flavors $Q, \widetilde{Q}$, N'_f-fundamental flavors $Q', \widetilde{Q}'$, N''_f-fundamental flavors $Q'', \widetilde{Q}''$, bifundamentals $F, \widetilde{F}$ and $G, \widetilde{G}$ can be described as two NS5-branes, two NS5'-branes, N_c-, N'_c- and N''_c-D4-branes, and N_f-, N'_f- and N''_f-D6-branes. The F is in the representation $(\mathbf{N_c}, \overline{\mathbf{N'_c}}, \mathbf{1})$ and the $\widetilde{F}$ is in the representation $(\overline{\mathbf{N_c}}, \mathbf{N'_c}, \mathbf{1})$ while the G is in the representation $(\mathbf{1}, \mathbf{N'_c}, \overline{\mathbf{N''_c}})$ and the $\widetilde{G}$ is in the representation $(\mathbf{1}, \overline{\mathbf{N'_c}}, \mathbf{N''_c})$, under the gauge group. The quarks Q and $\widetilde{Q}$ are in the representation $(\mathbf{N_c}, \mathbf{1}, \mathbf{1})$ and $(\overline{\mathbf{N_c}}, \mathbf{1}, \mathbf{1})$ respectively, the quarks Q' and $\widetilde{Q}'$ are in the representation $(\mathbf{1}, \mathbf{N'_c}, \mathbf{1})$ and $(\mathbf{1}, \overline{\mathbf{N'_c}}, \mathbf{1})$ respectively, and the quarks Q'' and $\widetilde{Q}''$ are in the representation $(\mathbf{1}, \mathbf{1}, \mathbf{N''_c})$ and $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{N''_c}})$ respectively, under the gauge group.

The mass terms for each fundamental quarks can be added by displacing each D6-branes along v direction leading to their coordinates $v = +v_{D6_{-\theta}}(+v_{D6_{-\theta'}})[+v_{D6_{-\theta''}}]$ respectively while the quartic terms for each fundamental quarks can be added also by rotating each D6-branes by an angle $-\theta(-\theta')[-\theta'']$ in (w, v)-plane respectively. Then, the general superpotential by

adding the above deformations is given by

$$\begin{aligned} W_{elec} &= \left[\beta_1 \operatorname{tr}(F\widetilde{F})^2 + \beta_2 \operatorname{tr}(F\widetilde{F}G\widetilde{G}) + \beta_3 \operatorname{tr}(G\widetilde{G})^2\right] + \frac{\alpha}{2}\operatorname{tr}(Q\widetilde{Q})^2 - m \operatorname{tr} Q\widetilde{Q} \\ &+ \frac{\alpha'}{2}\operatorname{tr}(Q'\widetilde{Q}')^2 - m' \operatorname{tr} Q'\widetilde{Q}' + \frac{\alpha''}{2}\operatorname{tr}(Q''\widetilde{Q}'')^2 - m'' \operatorname{tr} Q''\widetilde{Q}'' \end{aligned} \tag{4.1}$$

where the extra parameters are similarly described as the following geometric quantities

$$\alpha'' \equiv \frac{\tan\theta''}{\Lambda''}, \qquad m'' \equiv \frac{v_{D6_{-\theta''}}}{2\pi \ell_s^2}.$$

The first three terms of (4.1), in general, are due to the rotation angles ω_1 and ω_3 of the first and third NS-brane with respect to the second NS-brane and the rotation angles ω_2 and ω_4 of the second and fourth NS-brane with respect to the third NS-brane. We consider the limit where $\omega_1 = \omega_3 = \frac{\pi}{2} = \omega_2 = \omega_4$ and $\beta_i (i = 1, 2, 3)$ will vanish. Although the relative displacement of each two color D4-branes can be added in the superpotential, we focus on the particular limit $m_F = 0 = m_G$.

Then the $\mathcal{N} = 1$ supersymmetric electric brane configuration for the superpotential (4.1) in type IIA string theory is given as follows and let us draw this brane structure in Figure 4 explicitly:

- Two NS5-branes in (012345) directions
- Two NS5'-branes in (012389) directions
- N_f $D6_{-\theta}$-branes in (01237) directions and two other directions in (v, w)-plane
- N'_f $D6_{-\theta'}$-branes in (01237) directions and two other directions in (v, w)-plane
- N''_f $D6_{-\theta''}$-branes in (01237) directions and two other directions in (v, w)-plane
- N_c-, N'_c- and N''_c-color D4-branes in (01236) directions

4.2 Magnetic theory

The $NS5'_R$-brane starts out with linking number $l_e = \frac{N_f}{2} - N''_c$ from Figure 4 and after duality this $NS5'_R$-brane ends up with linking number $l_m = -\frac{N_f}{2} + \widetilde{N}_c - N'_f - N''_f$ from Figure 5. We consider only the particular brane motion where N_f $D6_{-\theta}$-branes meet the $NS5'_L$-brane and the $NS5_R$-brane with *no angles*. That is, the $D6_{-\theta}$-branes become D6-branes when they meet with the $NS5'_L$-brane instantaneously and then after that they come back to the original $D6_{-\theta}$-branes. Moreover, these $D6_{-\theta}$-branes become $D6_{-\frac{\pi}{2}}$-branes when they meet with the $NS5_R$-brane instantaneously and then after that they come back to the original $D6_{-\theta}$-branes. Therefore, in this dual process, there is *no* creation of D4-branes. Of course, the $D6_{-\theta}$-branes meet the $NS5'_R$-brane with an angle. Similarly, the N'_f $D6_{-\theta'}$-branes meet the $NS5'_L$-brane

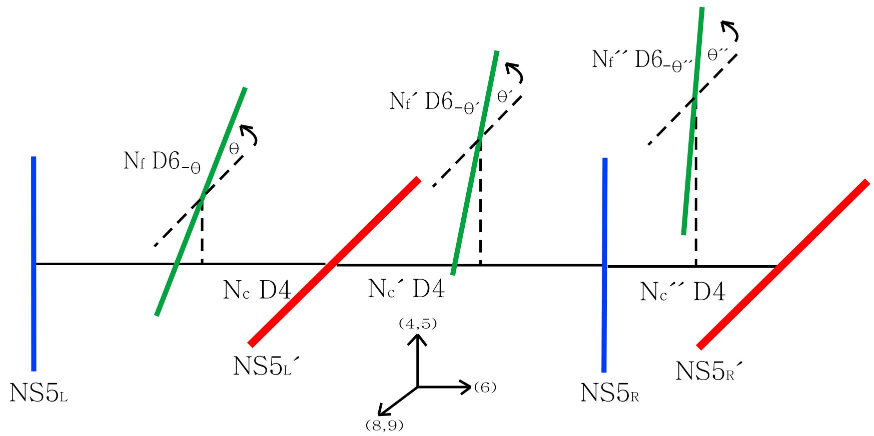

Figure 4: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N_c') \times SU(N_c'')$ with bifundamentals $F, \widetilde{F}, G, \widetilde{G}$, and fundamentals $Q, \widetilde{Q}, Q', \widetilde{Q}', Q'', \widetilde{Q}''$. A rotation of $N_f(N_f')[N_f'']$ D6-branes in (w, v)-plane corresponds to a quartic term for the fundamentals $Q, \widetilde{Q}(Q', \widetilde{Q}')[Q'', \widetilde{Q}'']$ while a displacement of $N_f(N_f')[N_f'']$ D6-branes in $+v$ direction corresponds to a mass term for the fundamentals $Q, \widetilde{Q}(Q', \widetilde{Q}')[Q'', \widetilde{Q}'']$. Let us take $v_{D6_{-\theta}} < v_{D6_{-\theta'}} < v_{D6_{-\theta''}}$ and $\theta < \theta' < \theta''$.

with no angles and the N_f'' $D6_{-\theta''}$-branes meet the $NS5_L'$-brane and the $NS5_R$-brane with no angles. Then the dual color number $\widetilde{N}_c$ is given by $\widetilde{N}_c = N_f + N_f' + N_f'' - N_c''$.

The $NS5_R$-brane starts out with linking number $l_e = N_c'' - N_c'$ and after duality this $NS5_R$-brane ends up with linking number $l_m = \widetilde{N}_c' - \widetilde{N}_c$. As we observed above, we consider only the particular brane motion where all the $D6_{-\theta,-\theta',-\theta''}$-branes become $D6_{-\frac{\pi}{2}}$-branes when they meet with the $NS5_R$-brane instantaneously and after that they come back to the original $D6_{-\theta,-\theta',-\theta''}$-branes. Therefore, in this dual process, there is no creation of D4-branes. Then it turns out that the dual color number $\widetilde{N}_c'$ is given by $\widetilde{N}_c' = N_f + N_f' + N_f'' - N_c'$.

The $NS5_L$-brane starts out with linking number $l_e = N_c - \frac{(N_f'+N_f'')}{2}$ and after duality this $NS5_L$-brane ends up with linking number $l_m = -\widetilde{N}_c'' + N_f + \frac{(N_f'+N_f'')}{2}$. As we observed above, we consider only the particular brane motion where all the $D6_{-\theta',-\theta''}$-branes meet the $NS5_L$-brane *with angles*. Then it turns out that the dual color number $\widetilde{N}_c''$ is given by $\widetilde{N}_c'' = N_f + N_f' + N_f'' - N_c$.

Then one obtains the folllowing dual color numbers

$$\widetilde{N}_c = N_f + N_f' + N_f'' - N_c'', \qquad \widetilde{N}_c' = N_f + N_f' + N_f'' - N_c', \qquad \widetilde{N}_c'' = N_f + N_f' + N_f'' - N_c.$$

The low energy theory [12, 15] on the three color D4-branes has $SU(\widetilde{N}_c) \times SU(\widetilde{N}_c') \times SU(\widetilde{N}_c'')$ gauge group and N_f-fundamental dual quarks $q'', \widetilde{q}''$, N_f'-fundamental dual quarks $q', \widetilde{q}'$, N_f''-fundamental dual quarks $q, \widetilde{q}$, bifundamentals $f, \widetilde{f}, g, \widetilde{g}$ and various gauge singlets.

The f is in the representation $(\widetilde{\mathbf{N_c}}, \overline{\widetilde{\mathbf{N'_c}}}, \mathbf{1})$ while the $\widetilde{f}$ is in the representation $(\overline{\widetilde{\mathbf{N_c}}}, \widetilde{\mathbf{N'_c}}, \mathbf{1})$ and the g is in the representation $(\mathbf{1}, \widetilde{\mathbf{N'_c}}, \overline{\widetilde{\mathbf{N''_c}}})$ while the $\widetilde{g}$ is in the representation $(\mathbf{1}, \overline{\widetilde{\mathbf{N'_c}}}, \widetilde{\mathbf{N''_c}})$, under the dual gauge group. The N''_f flavors q and $\widetilde{q}$ are in the representation $(\widetilde{\mathbf{N_c}}, \mathbf{1}, \mathbf{1})$ and $(\overline{\widetilde{\mathbf{N_c}}}, \mathbf{1}, \mathbf{1})$ respectively under the gauge group and in the representation $(\overline{\mathbf{N''_f}}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{N''_f})$ respectively under the flavor group $SU(N''_f)_L \times SU(N''_f)_R$. Similarly, the N'_f flavors q' and $\widetilde{q}'$ are in the representation $(\mathbf{1}, \widetilde{\mathbf{N'_c}}, \mathbf{1})$ and $(\mathbf{1}, \overline{\widetilde{\mathbf{N'_c}}}, \mathbf{1})$ respectively under the gauge group and in the representation $(\overline{\mathbf{N'_f}}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{N'_f})$ respectively under the flavor group $SU(N'_f)_L \times SU(N'_f)_R$. The N_f flavors q'' and $\widetilde{q}''$ are in the representation $(\mathbf{1}, \mathbf{1}, \widetilde{\mathbf{N''_c}})$ and $(\mathbf{1}, \mathbf{1}, \overline{\widetilde{\mathbf{N''_c}}})$ respectively under the gauge group and in the representation $(\overline{\mathbf{N_f}}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{N_f})$ respectively under the flavor group $SU(N_f)_L \times SU(N_f)_R$.

In particular, a magnetic meson field $M_0 \equiv Q\widetilde{Q}$ is $N_f \times N_f$ matrix and comes from 4-4 strings of N_f flavor D4-branes created when N_f $D6_{-\theta}$-branes meet the $NS5'_R$-brane, a magnetic meson field $M'_0 \equiv Q'\widetilde{Q}'$ is $N'_f \times N'_f$ matrix and comes from 4-4 strings of N'_f flavor D4-branes created when N'_f $D6_{-\theta'}$-branes meet the $NS5_L$-brane and a magnetic meson field $M''_0 \equiv Q''\widetilde{Q}''$ is $N''_f \times N''_f$ matrix and comes from 4-4 strings of N''_f flavor D4-branes created when N''_f $D6_{-\theta''}$-branes meet the $NS5_L$-brane.

Then the most general magnetic superpotential, for the case where $N_f(N'_f)[N''_f]$ $D6_{-\theta}$-branes($D6_{-\theta'}$-branes)[$D6_{-\theta''}$-branes] meet the NS-branes *with angles*, is given by

$$\begin{aligned}
W_{dual} &= \left[(f\widetilde{f})^2 + f\widetilde{f}g\widetilde{g} + (g\widetilde{g})^2\right] \\
&+ \left(\frac{\alpha}{2}\operatorname{tr} M_0^2 - mM_0\right) + \left(\frac{\alpha'}{2}\operatorname{tr} M_0'^2 - m'M'_0\right) + \left(\frac{\alpha''}{2}\operatorname{tr} M_0''^2 - m''M''_0\right) \\
&+ \left[M_0\widetilde{q}''(f\widetilde{f})^2 q'' + M'_0\widetilde{q}'(f\widetilde{f})^2 q' + M''_0\widetilde{q}(f\widetilde{f})^2 q\right] \\
&+ \Big[M_2\widetilde{q}''f\widetilde{f}q'' + M_4\widetilde{q}''q'' + M'_{2,F}\widetilde{q}'g\widetilde{g}q' + M'_{2,G}\widetilde{q}'f\widetilde{f}q' + M'_4\widetilde{q}'q' + M''_2\widetilde{q}f\widetilde{f}q \\
&+ M''_4\widetilde{q}q + P_1\widetilde{q}'f\widetilde{f}f\widetilde{q}'' + \widetilde{P}_1 q'\widetilde{f}f\widetilde{f}q'' + P_2\widetilde{q}f\widetilde{g}q'' + \widetilde{P}_2 q\widetilde{f}g\widetilde{q}'' + P_3\widetilde{q}'f\widetilde{q}'' \\
&+ \widetilde{P}_3 q'\widetilde{f}q'' + R_1\widetilde{q}'f\widetilde{f}f\widetilde{q} + \widetilde{R}_1 q'\widetilde{f}f\widetilde{f}q + R_3\widetilde{q}'f\widetilde{q} + \widetilde{R}_3 q'\widetilde{f}q\Big]
\end{aligned} \tag{4.2}$$

where the mesons are given by [12, 15]

$$\begin{aligned}
M_0 &\equiv Q\widetilde{Q}, \quad M'_0 \equiv Q'\widetilde{Q}', \quad M''_0 \equiv Q''\widetilde{Q}'', \quad M_2 \equiv Q\widetilde{F}F\widetilde{Q}, \quad M_4 \equiv Q(\widetilde{F}F)^2\widetilde{Q}, \\
M'_{2,F} &\equiv Q'\widetilde{F}F\widetilde{Q}', \quad M'_{2,G} \equiv Q'\widetilde{G}G\widetilde{Q}', \quad M'_4 \equiv Q'(\widetilde{F}F)^2\widetilde{Q}', \quad M''_2 \equiv Q''\widetilde{G}G\widetilde{Q}'', \\
M''_4 &\equiv Q''(\widetilde{G}G)^2\widetilde{Q}'', \quad P_1 \equiv Q\widetilde{F}Q', \quad \widetilde{P}_1 \equiv \widetilde{Q}F\widetilde{Q}', \quad P_2 \equiv Q\widetilde{F}G\widetilde{Q}'', \\
\widetilde{P}_2 &\equiv \widetilde{Q}F\widetilde{G}Q'', \quad P_3 \equiv Q\widetilde{F}F\widetilde{F}Q', \quad \widetilde{P}_3 \equiv \widetilde{Q}F\widetilde{F}F\widetilde{Q}', \quad R_1 \equiv Q'\widetilde{G}Q'', \\
\widetilde{R}_1 &\equiv \widetilde{Q}'G\widetilde{Q}'', \quad R_3 \equiv Q'\widetilde{G}G\widetilde{G}Q'', \quad \widetilde{R}_3 \equiv \widetilde{Q}'G\widetilde{G}G\widetilde{Q}''.
\end{aligned}$$

The first two lines of (4.2) are dual expressions for the electric superpotential (4.1) and the corresponding meson fields M_0, M'_0, M''_0 are replaced and the remaining lines of (4.2) are the analogs of the cubic term superpotential between the meson and dual quarks in Seiberg duality.

When the N_f $D6_{-\theta}$-branes meet the $NS5'_L$-brane and $NS5_R$-brane, no creation of D4-branes implies that there is no M_2- or M_4-term in the above superpotential (4.2). The mesons M_2 and M_4 originate from $SU(N_c)$ chiral mesons $Q\widetilde{Q}$ when one dualizes the $SU(N_c)$ gauge group first by moving the $NS5'_L$-brane to the left of the $NS5_L$-brane. That is, the fluctuations of strings stretching between the N_f "flavor" D4-branes provide these meson fields. After the additional dual procedures, the cubic terms in the superpotential arise as M_2-dependent and M_4-dependent terms where M_2 has extra $\widetilde{F}F$ fields and M_4 has an extra $(\widetilde{F}F)^2$ fields, besides $Q\widetilde{Q}$, due to the further dualizations. The M_2-term in the superpotential has an extra $f\widetilde{f}$ factor besides $\widetilde{q}''q''$.

When the N''_f $D6_{-\theta''}$-branes meet the $NS5'_L$-brane and $NS5_R$-brane, no creation of D4-branes implies that there is no M''_2- or M''_4-term in the above superpotential (4.2). The mesons M''_2 and M''_4 originate from $SU(N''_c)$ chiral mesons $Q''\widetilde{Q}''$ when one dualizes the $SU(N''_c)$ gauge group first by moving the $NS5_R$-brane to the right of the $NS5'_R$-brane. That is, the fluctuations of strings stretching between the N''_f "flavor" D4-branes provide these meson fields. After the additional dual procedures, the cubic terms in the superpotential arise as M''_2-dependent and M''_4-dependent terms where M''_2 has extra $\widetilde{G}G$ fields and M''_4 has extra $(\widetilde{G}G)^2$ fields, besides $Q''\widetilde{Q}''$, due to the further dualizations. The M''_2-term in the superpotential has an extra $f\widetilde{f}$ factor besides $\widetilde{q}q$.

Similarly, when the N'_f $D6_{-\theta'}$-branes meet the $NS5'_L$-brane, the $NS5_R$-brane, or $NS5'_R$-brane with no angles, there is no M'_4 term in the above superpotential (4.2) [1]. These meson fields originate from $SU(N'_c)$ chiral mesons $Q'\widetilde{Q}'$ when one dualizes the $SU(N'_c)$ gauge group first by interchanging the $NS5'_L$-brane and the $NS5_R$-brane each other. The strings stretching between the N'_f "flavor" D4-branes provide this meson. After the additional dual procedures, the cubic terms in the superpotential arise as $M'_{2,F}$-term, $M'_{2,G}$-term and M'_4-term in (4.2) where $M'_{2,F}$ has extra $\widetilde{F}F$ fields, $M'_{2,G}$ has extra $\widetilde{G}G$ fields, and M'_4 has extra $(\widetilde{F}F)^2$ fields, besides $Q'\widetilde{Q}'$, due to the further dualizations. The $M'_{2,F}$-term in the superpotential has an extra $g\widetilde{g}$ factor while $M'_{2,G}$-term has an extra $f\widetilde{f}$ factor, besides $\widetilde{q}'q'$.

[1] In [15], they introduced the $2N'_f$ full D4-branes without changing the linking number in order to satisfy the correct dual color numbers in the gauge theory side analysis. This has led to the fact that there are $2N'_f$ D4-branes connecting the $NS5_R$-brane and N'_f $D6_{-\theta'}$-branes, after duality. See Figure 5. In other words, these extra $2N'_f$ D4-branes were needed for the existence of meson fields $M'_{2,F}$ and $M'_{2,G}$. In our construction, we do not need these extra $2N'_f$ full D4-branes because we do not want to have these unwanted meson fields $M'_{2,F}$ and $M'_{2,G}$.

Furthermore, when the N_f $D6_{-\theta}$-branes, the N'_f $D6_{-\theta'}$-branes, the $NS5'_L$-brane and the $NS5_R$-brane meet each other with no angles, no P_1- and P_3- or $\widetilde{P}_1$- and $\widetilde{P}_3$-dependent terms occur in the superpotential (4.2). These mesons originate from $SU(N'_c)$ chiral mesons $\widetilde{F}Q'$ and $F\widetilde{Q}'$ when one dualizes the $SU(N'_c)$ first by interchanging the $NS5'_L$-brane and the $NS5_R$-brane. The strings stretching between the N'_f flavor D4-branes and N_c color D4-branes give rise to these N'_f $SU(N_c)$ fundamentals and N'_f $SU(N_c)$ antifundamentals. After the additional dual procedures, these cubic terms arise as these meson terms where there exist extra $\widetilde{f}f\widetilde{q}'', f\widetilde{f}q'', \widetilde{q}''$ and q'' in the interactions of $P_1, \widetilde{P}_1, P_3$ and $\widetilde{P}_3$ in the superpotential and the mesons have extra $Q, \widetilde{Q}, Q\widetilde{F}F, \widetilde{Q}F\widetilde{F}$ fields respectively, due to the further dualizations.

When the N'_f $D6_{-\theta'}$-branes, the N''_f $D6_{-\theta''}$-branes, the $NS5'_L$-brane and the $NS5_R$-brane meet each other with no angles, no R_1- and R_3- or $\widetilde{R}_1$- and $\widetilde{R}_3$-dependent terms arise in the superpotential (4.2). These mesons originate from $SU(N''_c)$ chiral mesons $\widetilde{G}Q''$ and $G\widetilde{Q}''$ when one dualizes the $SU(N''_c)$ first by moving the $NS5_R$-brane to the right of the $NS5'_R$-brane. The strings stretching between the N''_f flavor D4-branes and N'_c color D4-branes give rise to these N''_f $SU(N'_c)$ fundamentals and N''_f $SU(N'_c)$ antifundamentals. After the additional dual procedures, these cubic terms arise as these meson terms where $R_1, \widetilde{R}_1, R_3$ and $\widetilde{R}_3$ have extra $Q', \widetilde{Q}', Q'\widetilde{G}G, \widetilde{Q}'G\widetilde{G}$ fields respectively, due to the further dualizations.

Finally, when the N_f $D6_{-\theta}$-branes, the N''_f $D6_{-\theta''}$-branes, the $NS5'_L$-brane and the $NS5_R$-brane meet each other with no angles, no P_2- or $\widetilde{P}_2$-dependent terms arise in the superpotential (4.2). These mesons originate from $SU(N_c)$ chiral mesons $Q\widetilde{F}$ and $\widetilde{Q}F$ when one dualizes the $SU(N_c)$ first by moving the $NS5'_L$-brane to the left of the $NS5_L$-brane. The strings stretching between the N_f flavor D4-branes and N'_c color D4-branes give rise to these N_f $SU(N'_c)$ fundamentals and N_f $SU(N'_c)$ antifundamentals. After the additional dual procedures, these cubic terms arise as these meson terms where there exist extra $\widetilde{g}q'', g\widetilde{q}''$ in the interactions of P_2 and $\widetilde{P}_2$ in the superpotential and the mesons have extra $G\widetilde{Q}'', \widetilde{G}Q''$ fields respectively, due to the further dualizations.

Then the reduced magnetic superpotential in our case by taking the first three lines of (4.2) is given by

$$\begin{aligned} W_{dual} &= \left[M_0\widetilde{q}''(f\widetilde{f})^2q'' + \frac{\alpha}{2}\,\mathrm{tr}\, M_0^2 - mM_0\right] + \left[M'_0\widetilde{q}'(f\widetilde{f})^2q' + \frac{\alpha'}{2}\,\mathrm{tr}\, {M'_0}^2 - m'M'_0\right] \\ &+ \left[M''_0\widetilde{q}(f\widetilde{f})^2q + \frac{\alpha''}{2}\,\mathrm{tr}\, {M''_0}^2 - m''M''_0\right]. \end{aligned} \tag{4.3}$$

For the supersymmetric vacua, one can compute the F-term equations for this superpo-

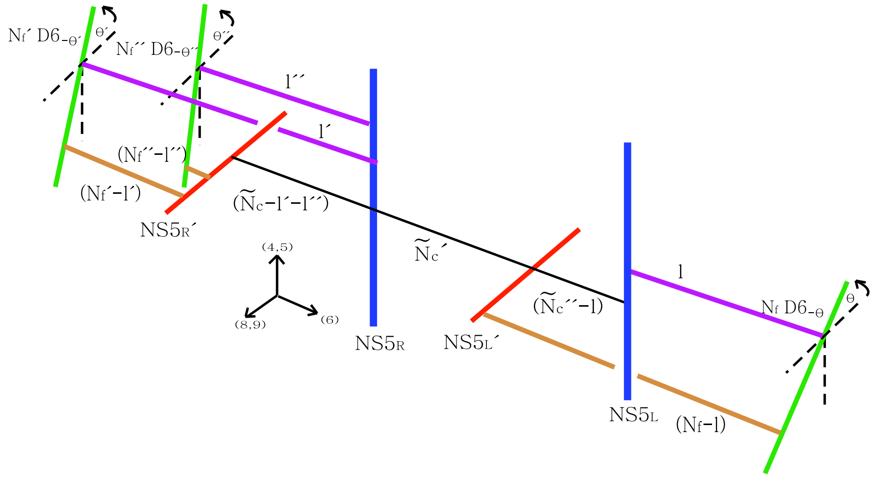

Figure 5: The $\mathcal{N}=1$ supersymmetric magnetic brane configuration corresponding to Figure 4 with a splitting and a reconnection between D4-branes when the gravitational potential of the NS5-brane is ignored. The N_f flavor D4-branes connecting between $D6_{-\theta}$-branes and $NS5'_L$-brane are splitting into (N_f-l)- and l- D4-branes, the N'_f flavor D4-branes connecting between $D6_{-\theta'}$-branes and $NS5'_R$-brane are splitting into (N'_f-l')- and l'- D4-branes, and the N''_f flavor D4-branes connecting between $D6_{-\theta''}$-branes and $NS5'_R$-brane are splitting into (N''_f-l'')- and l''- D4-branes.

tential (4.3) and the F-terms for $M_0, q'', \widetilde{q}'', M'_0, q', \widetilde{q}', M''_0, q, \widetilde{q}, f$ and $\widetilde{f}$ are given by

$$\begin{aligned}
&\widetilde{q}''(f\widetilde{f})^2 q'' - m + \alpha M_0 = 0, \qquad (M_0\widetilde{q}''f\widetilde{f})f\widetilde{f} = 0, \qquad f\widetilde{f}(f\widetilde{f}q''M_0) = 0,\\
&\widetilde{q}'(f\widetilde{f})^2 q' - m' + \alpha' M'_0 = 0, \qquad (M'_0\widetilde{q}'f\widetilde{f})f\widetilde{f} = 0, \qquad f\widetilde{f}(f\widetilde{f}q'M'_0) = 0,\\
&\widetilde{q}(f\widetilde{f})^2 q - m'' + \alpha'' M''_0 = 0, \qquad (M''_0\widetilde{q}f\widetilde{f})f\widetilde{f} = 0, \qquad f\widetilde{f}(f\widetilde{f}qM''_0) = 0,\\
&\widetilde{f}(f\widetilde{f}q''M_0)\widetilde{q}'' + \widetilde{f}q''(M_0\widetilde{q}''f\widetilde{f}) + \widetilde{f}(f\widetilde{f}q'M'_0)\widetilde{q}'\\
&+\widetilde{f}q'(M'_0\widetilde{q}'f\widetilde{f}) + \widetilde{f}(f\widetilde{f}qM''_0)\widetilde{q} + \widetilde{f}q(M''_0\widetilde{q}f\widetilde{f}) = 0,\\
&(f\widetilde{f}q''M_0)\widetilde{q}''f + q''(M_0\widetilde{q}''f\widetilde{f})f + (f\widetilde{f}q'M'_0)\widetilde{q}'f\\
&+q'(M'_0\widetilde{q}'f\widetilde{f})f + (f\widetilde{f}qM''_0)\widetilde{q}f + q'(M''_0\widetilde{q}f\widetilde{f})f = 0.
\end{aligned}$$

From this, it is easy to see that the last two equations are satisfied if the second, third, fifth, sixth, eighth, and ninth are satisfied:$M_0\widetilde{q}''f\widetilde{f} = 0 = \cdots = f\widetilde{f}qM''_0$.

The theory has many nonsupersymmetric meta-stable ground states and when we rescale the meson fields as $M_0 = h\Lambda\Phi_0, M'_0 = h'\Lambda'\Phi'_0$ and $M''_0 = h''\Lambda''\Phi''_0$, then the Kahler potential for Φ_0, Φ'_0 and Φ''_0 is canonical and the magnetic quarks are canonical near the origin of field

space. Then the magnetic superpotential (4.3) can be rewritten as

$$
\begin{aligned}
W_{mag} &= \left[h\Phi_0\widetilde{q}''(f\widetilde{f})^2q'' + \frac{h^2\mu_\phi}{2}\,\mathrm{tr}\,\Phi_0^2 - h\mu^2\,\mathrm{tr}\,\Phi_0\right] + \left[h'\Phi_0'\widetilde{q}'(f\widetilde{f})^2q' + \frac{h'^2\mu_\phi'}{2}\,\mathrm{tr}\,\Phi_0'^2 - h'\mu'^2\,\mathrm{tr}\,\Phi_0'\right] \\
&+ \left[h''\Phi_0''\widetilde{q}(f\widetilde{f})^2q + \frac{h''^2\mu_\phi''}{2}\,\mathrm{tr}\,\Phi_0''^2 - h''\mu''^2\,\mathrm{tr}\,\Phi_0''\right]
\end{aligned}
$$

where $\mu^2 = m\Lambda, \mu'^2 = m'\Lambda', \mu''^2 = m''\Lambda''$ and $\mu_\phi = \alpha\Lambda^2, \mu_\phi' = \alpha'\Lambda'^2, \mu_\phi'' = \alpha''\Lambda''^2$.

Now one splits the $(N_f - l) \times (N_f - l)$ block at the lower right corner of $h\Phi_0$ and $\widetilde{q}''(f\widetilde{f})^2q''$ into blocks of size n and $(N_f - l - n)$, one decomposes the $(N_f' - l') \times (N_f' - l')$ block at the lower right corner of $h'\Phi_0'$ and $\widetilde{q}'(f\widetilde{f})^2q'$ into blocks of size n' and $(N_f' - l' - n')$ and one splits the $(N_f'' - l'') \times (N_f'' - l'')$ block at the lower right corner of $h''\Phi_0''$ and $\widetilde{q}(f\widetilde{f})^2q$ into blocks of size n'' and $(N_f'' - l'' - n'')$ as follows [3]:

$$
\begin{aligned}
h\Phi_0 &= \begin{pmatrix} 0_l & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi}\mathbf{1}_{N_f-l-n} \end{pmatrix}, h'\Phi_0' = \begin{pmatrix} 0_{l'} & 0 & 0 \\ 0 & h'\Phi_{n'}' & 0 \\ 0 & 0 & \frac{\mu'^2}{\mu_\phi'}\mathbf{1}_{N_f'-l'-n'} \end{pmatrix}, \qquad (4.4) \\
h''\Phi_0'' &= \begin{pmatrix} 0_{l''} & 0 & 0 \\ 0 & h''\Phi_{n''}'' & 0 \\ 0 & 0 & \frac{\mu''^2}{\mu_\phi''}\mathbf{1}_{N_f''-l''-n''} \end{pmatrix}, \widetilde{q}''(f\widetilde{f})^2q'' = \begin{pmatrix} \mu^2\mathbf{1}_l & 0 & 0 \\ 0 & \widetilde{\varphi}''(y\widetilde{y})^2\varphi'' & 0 \\ 0 & 0 & 0_{N_f-l-n} \end{pmatrix}, \\
\widetilde{q}'(f\widetilde{f})^2q' &= \begin{pmatrix} \mu'^2\mathbf{1}_{l'} & 0 & 0 \\ 0 & \widetilde{\varphi}'(y\widetilde{y})^2\varphi' & 0 \\ 0 & 0 & 0_{N_f'-l'-n'} \end{pmatrix}, \widetilde{q}(f\widetilde{f})^2q = \begin{pmatrix} \mu''^2\mathbf{1}_{l''} & 0 & 0 \\ 0 & \widetilde{\varphi}(y\widetilde{y})^2\varphi & 0 \\ 0 & 0 & 0_{N_f''-l''-n''} \end{pmatrix}.
\end{aligned}
$$

Here φ'' and $\widetilde{\varphi}''$ are $n \times (\widetilde{N}_c'' - l)$ dimensional matrices, φ' and $\widetilde{\varphi}'$ are $n' \times (\widetilde{N}_c' - l')$ dimensional matrices and φ and $\widetilde{\varphi}$ are $n'' \times (\widetilde{N}_c - l'')$ dimensional matrices. In the brane configuration shown in Figure 6, φ'' and $\widetilde{\varphi}''$ correspond to fundamental strings connecting the n flavor D4-branes and $(\widetilde{N}_c'' - l)$ color D4-branes, φ' and $\widetilde{\varphi}'$ correspond to fundamental strings connecting the n' flavor D4-branes and $(\widetilde{N}_c' - l')$ color D4-branes and φ and $\widetilde{\varphi}$ correspond to fundamental strings connecting the n'' flavor D4-branes and $(\widetilde{N}_c - l'')$ color D4-branes. The Φ_n and $\widetilde{\varphi}''(y\widetilde{y})^2\varphi''$ are $n \times n$ matrices, $\Phi_{n'}'$ and $\widetilde{\varphi}'(y\widetilde{y})^2\varphi'$ are $n' \times n'$ matrices and $\Phi_{n''}''$ and $\widetilde{\varphi}(y\widetilde{y})^2\varphi$ are $n'' \times n''$ matrices.

The supersymmetric ground state corresponds to $h\Phi_n = \frac{\mu^2}{\mu_\phi}\mathbf{1}_n, \widetilde{\varphi}''y\widetilde{y} = 0 = y\widetilde{y}\varphi''$, $h'\Phi_{n'}' = \frac{\mu'^2}{\mu_\phi'}\mathbf{1}_{n'}, \widetilde{\varphi}'y\widetilde{y} = 0 = y\widetilde{y}\varphi'$ and $h''\Phi_{n''}'' = \frac{\mu''^2}{\mu_\phi''}\mathbf{1}_{n''}, \widetilde{\varphi}y\widetilde{y} = 0 = y\widetilde{y}\varphi$. The l of the N_f-flavor D4-branes are reconnected with l-color D4-branes and the resulting l D4-branes stretch from the $D6_{-\theta}$-branes to the $NS5_L$-brane directly and the intersection point between the l D4-branes and the $D6_{-\theta}$-branes is given by $(v, w) = (+v_{D6_{-\theta}}, 0)$. This corresponds to exactly the l's

eigenvalues from zeros of $h\Phi_0$ in (4.4). Now the remaining $(N_f - l)$-flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5'_L$-brane correspond to the eigenvalues of $h\Phi_0$ in (4.4), i.e., $\frac{\mu^2}{\mu_\phi}\mathbf{1}_{N_f-l}$. The intersection point between the $(N_f - l)$ D4-branes and the $NS5'_L$-branes is given by $(v, w) = (0, +v_{D6_{-\theta}} \cot\theta)$ from trigonometric geometry.

Similarly, the l' of the N'_f-flavor D4-branes are reconnected with l'-color D4-branes and the resulting l' D4-branes stretch from the $D6_{-\theta'}$-branes to the $NS5_R$-brane directly and the intersection point between the l' D4-branes and the $D6_{-\theta'}$-branes is given by $(v, w) = (+v_{D6_{-\theta'}}, 0)$. This corresponds to exactly the l''s eigenvalues from zeros of $h'\Phi'_0$ in (4.4). Now the remaining $(N'_f - l')$-flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5'_R$-brane correspond to the eigenvalues of $h'\Phi'_0$ in (4.4), i.e., $\frac{\mu'^2}{\mu'_\phi}\mathbf{1}_{N'_f-l'}$. The intersection point between the $(N'_f - l')$ D4-branes and the $NS5'_R$-branes is given by $(v, w) = (0, +v_{D6_{-\theta'}} \cot\theta')$ from trigonometric geometry.

The l'' of the N''_f-flavor D4-branes are reconnected with l''-color D4-branes and the resulting l'' D4-branes stretch from the $D6_{-\theta''}$-branes to the $NS5_R$-brane directly and the intersection point between the l'' D4-branes and the $D6_{-\theta''}$-branes is given by $(v, w) = (+v_{D6_{-\theta''}}, 0)$. This corresponds to exactly the l'''s eigenvalues from zeros of $h''\Phi''_0$ in (4.4). Now the remaining $(N''_f - l'')$-flavor D4-branes between the $D6_{-\theta''}$-branes and the $NS5'_R$-brane correspond to the eigenvalues of $h''\Phi''_0$ in (4.4), i.e., $\frac{\mu''^2}{\mu''_\phi}\mathbf{1}_{N''_f-l''}$. The intersection point between the $(N''_f - l'')$ D4-branes and the $NS5'_R$-branes is given by $(v, w) = (0, +v_{D6_{-\theta''}} \cot\theta'')$ from trigonometric geometry.

Now the full one loop potential containing $\Phi_n, \Phi'_{n'}, \Phi''_{n''}, \Phi_n^\dagger, {\Phi'}^\dagger_{n'}$ and ${\Phi''}^\dagger_{n''}$ with $\mu_\phi << \mu << \Lambda_m$, $\mu'_\phi << \mu' << \Lambda'_m$ and $\mu''_\phi << \mu'' << \Lambda''_m$, by combining the superpotential and the vacuum expectation values for the fields, takes the form

$$
\begin{aligned}
V &= |h\Phi_n\widetilde{\varphi}'' y\widetilde{y}|^2 + |hy\widetilde{y}\varphi''\Phi_n|^2 + |h\widetilde{\varphi}''(y\widetilde{y})^2\varphi'' - h\mu^2\mathbf{1}_n + h^2\mu_\phi\Phi_n|^2 \\
&+ |h'\Phi'_{n'}\widetilde{\varphi}' y\widetilde{y}|^2 + |h'y\widetilde{y}\varphi'\Phi'_{n'}|^2 + |h'\widetilde{\varphi}'(y\widetilde{y})^2\varphi' - h'\mu'^2\mathbf{1}_{n'} + h'^2\mu'_\phi\Phi'_{n'}|^2 \\
&+ |h''\Phi''_{n''}\widetilde{\varphi} y\widetilde{y}|^2 + |h''y\widetilde{y}\varphi\Phi''_{n''}|^2 + |h''\widetilde{\varphi}(y\widetilde{y})^2\varphi - h''\mu''^2\mathbf{1}_{n''} + h''^2\mu''_\phi\Phi''_{n''}|^2 \\
&+ b|h^2\mu|^2 \,\mathrm{tr}\, \Phi_n^\dagger\Phi_n + b'|h'^2\mu'|^2 \,\mathrm{tr}\, {\Phi'}^\dagger_{n'}\Phi'_{n'} + b''|h''^2\mu''|^2 \,\mathrm{tr}\, {\Phi''}^\dagger_{n''}\Phi''_{n''}
\end{aligned}
$$

where $b = \frac{(\ln 4-1)}{8\pi^2}\widetilde{N}_c$, $b' = \frac{(\ln 4-1)}{8\pi^2}\widetilde{N}'_c$ and $b'' = \frac{(\ln 4-1)}{8\pi^2}\widetilde{N}''_c$. Differentiating this potential with respect to $\Phi_n^\dagger$, ${\Phi'}^\dagger_{n'}$ and ${\Phi''}^\dagger_{n''}$ and putting $\widetilde{\varphi}'' y\widetilde{y} = 0 = y\widetilde{y}\varphi''$, $\widetilde{\varphi}' y\widetilde{y} = 0 = y\widetilde{y}\varphi'$ and $\widetilde{\varphi} y\widetilde{y} = 0 =$

$y\widetilde{y}\varphi$, one obtains

$$
\begin{aligned}
h\Phi_n &\simeq \frac{\mu_\phi}{b}\mathbf{1}_n \qquad \text{or} \qquad M_n \simeq \frac{\alpha\Lambda^3}{\widetilde{N}_c}\mathbf{1}_n, \\
h'\Phi'_{n'} &\simeq \frac{\mu'_\phi}{b'}\mathbf{1}_{n'} \qquad \text{or} \qquad M'_{n'} \simeq \frac{\alpha'\Lambda'^3}{\widetilde{N}'_c}\mathbf{1}_{n'}, \\
h''\Phi''_{n''} &\simeq \frac{\mu''_\phi}{b''}\mathbf{1}_{n''} \qquad \text{or} \qquad M''_{n''} \simeq \frac{\alpha''\Lambda''^3}{\widetilde{N}''_c}\mathbf{1}_{n''},
\end{aligned}
$$

corresponding to the w coordinates of n curved flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5'_L$-brane, the w coordinates of n' curved flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5'_R$-brane, the w coordinates of n'' curved flavor D4-branes between the $D6_{-\theta''}$-branes and the $NS5'_R$-brane respectively. Since $\frac{\mu_\phi}{b} << \frac{\mu^2}{\mu_\phi}$, $\frac{\mu'_\phi}{b'} << \frac{\mu'^2}{\mu'_\phi}$ and $\frac{\mu''_\phi}{b''} << \frac{\mu''^2}{\mu''_\phi}$, the n-, n'- and n''- curved D4-branes are nearer to $w = 0$ at which the $NS5_L$-brane or the $NS5_R$-brane is located.

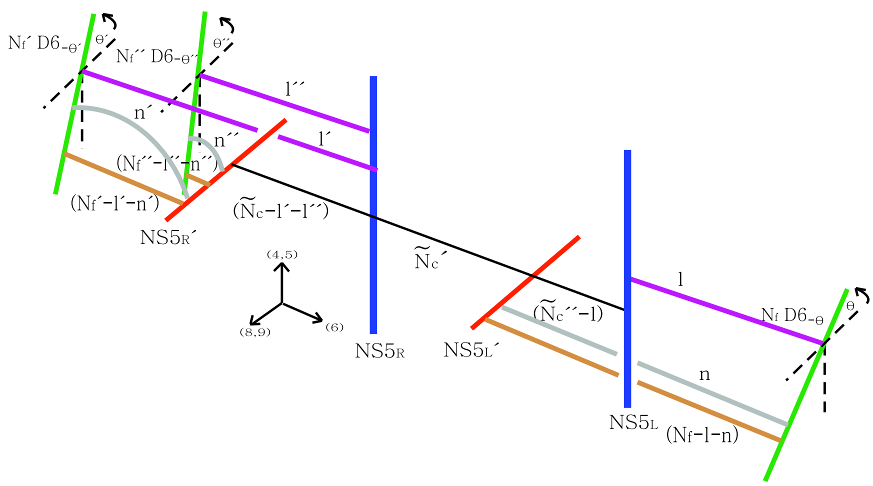

Figure 6: The nonsupersymmetric meta-stable magnetic brane configuration corresponding to Figure 4 with a misalignment between D4-branes when the gravitational potential of the NS5-brane is considered. The $(N_f - l)$ flavor D4-branes in Figure 5 connecting between $D6_{-\theta}$-branes and $NS5'_L$-brane are further splitting into $(N_f - l - n)$- and n- D4-branes, the $(N'_f - l')$ flavor D4-branes connecting between $D6_{-\theta'}$-branes and $NS5'_R$-brane are further splitting into $(N'_f - l' - n')$- and n'- D4-branes, and the $(N''_f - l'')$ flavor D4-branes connecting between $D6_{-\theta''}$-branes and $NS5'_R$-brane are further splitting into $(N''_f - l'' - n'')$- and n''- D4-branes. The shape of n "curved" D4-branes is more clear when the $D6_{-\theta}$-branes are moved into the left hand side of $NS5_L$-brane, as in [17].

4.3 Addition of adjoint fields

When we consider higher order superpotential for the adjoint fields whose degree is greater than four(i.e., the number of NS5-branes or NS5'-branes $k > 3$), there exist more meson fields. For odd k, the chiral ring truncates [15]. In general, it is not known how the deformations for the meson fields $QX_1^j\widetilde{Q}, QX_2^j\widetilde{Q}$ or $Q''X_3^j\widetilde{Q}''$ arise geometrically in the type IIA brane configuration. Therefore, it is not clear how to add these deformations in an electric theory or a magnetic theory.

5 $Sp(N_c)\times SO(2N_c')\times Sp(N_c'')$ with $2N_f$-fund., $2N_f'$-vectors, $2N_f''$-fund., and bifund.

Let us add the O4-plane to the brane configuration of previous section.

5.1 Electric theory

The type IIA supersymmetric electric brane configuration [31, 27] corresponding to $\mathcal{N}=1$ $Sp(N_c)\times SO(2N_c')\times Sp(N_c'')$ gauge theory with $2N_f$-fundamental fields Q, $2N_f'$-vectors Q', $2N_f''$-fundamental fields Q'', bifundamentals F and G can be described as two NS5-branes, two NS5'-branes, $2N_c$-, $2N_c'$- and $2N_c''$-D4-branes, $2N_f$-, $2N_f'$- and $2N_f''$-D6-branes and an orientifold 4-plane. The F is in the representation $(\mathbf{2N_c}, \mathbf{2N_c'}, \mathbf{1})$ while the G is in the representation $(\mathbf{1}, \mathbf{2N_c'}, \mathbf{2N_c''})$, under the gauge group. The quarks Q are in the representation $(\mathbf{2N_c}, \mathbf{1}, \mathbf{1})$, the quarks Q' are in the representation $(\mathbf{1}, \mathbf{2N_c'}, \mathbf{1})$, the quarks Q'' are in the representation $(\mathbf{1}, \mathbf{1}, \mathbf{2N_c''})$, under the gauge group.

The mass terms for each quarks can be added by displacing each D6-branes along v direction leading to their coordinates $v = \pm v_{D6_{-\theta}}(\pm v_{D6_{-\theta'}})[\pm v_{D6_{-\theta''}}]$ respectively while the quartic terms for each quarks can be added also by rotating each D6-branes by an angle $-\theta(-\theta')[-\theta'']$ in (w,v)-plane respectively. Then, the general superpotential by adding the above deformations is given by

$$\begin{aligned} W_{elec} &= \left[\beta_1 \,\mathrm{tr}\, F^4 + \beta_2 \,\mathrm{tr}(F^2G^2) + \beta_3 \,\mathrm{tr}\, G^4\right] + \frac{\alpha}{2}\,\mathrm{tr}(QQ)^2 - m\,\mathrm{tr}\, QQ \\ &+ \frac{\alpha'}{2}\,\mathrm{tr}(Q'Q')^2 - m'\,\mathrm{tr}\, Q'Q' + \frac{\alpha''}{2}\,\mathrm{tr}(Q''Q'')^2 - m''\,\mathrm{tr}\, Q''Q''. \end{aligned} \tag{5.1}$$

We consider the limit where $\omega_1 = \omega_3 = \frac{\pi}{2} = \omega_2 = \omega_4$ and $\beta_i (i = 1, 2, 3)$ will vanish. Although the relative displacement of each two color D4-branes can be added in the superpotential, we focus on the particular limit $m_F = 0 = m_G$. The mass matrix m is antisymmetric, the mass matrix m' is symmetric and the mass matrix m'' is antisymmetric.

Then the $\mathcal{N}=1$ supersymmetric electric brane configuration for the superpotential (5.1) in type IIA string theory is given as follows:

- Two NS5-branes in (012345) directions
- Two NS5'-branes in (012389) directions
- $2N_f$ $D6_{-\theta}$-branes in (01237) directions and two other directions in (v, w)-plane
- $2N'_f$ $D6_{-\theta'}$-branes in (01237) directions and two other directions in (v, w)-plane
- $2N''_f$ $D6_{-\theta''}$-branes in (01237) directions and two other directions in (v, w)-plane
- $2N_c$-, $2N'_c$- and $2N''_c$-color D4-branes in (01236) directions
- $O4^{\pm}$-planes in (01236) directions

The corresponding brane configuration can be obtained from the previous section by considering the correct mirrors based on the O4-plane action.

5.2 Magnetic theory

It is straightforward to compute the dual color numbers by considering the D4-brane charge of an orientifold 4-plane.

The $NS5'_R$-brane starts out with linking number $l_e = \frac{2N_f}{2} - 2N''_c - 2$ and after duality this $NS5'_R$-brane ends up with linking number $l_m = -\frac{2N_f}{2} + 2\widetilde{N}_c - 2N'_f - 2N''_f + 2$. We consider only the particular brane motion where N_f $D6_{-\theta}$-branes meet the $NS5'_L$-brane and the $NS5_R$-brane with *no* angles(and their mirrors). That is, the $D6_{-\theta}$-branes become D6-branes when they meet with the $NS5'_L$-brane instantaneously and then after that they come back to the original $D6_{-\theta}$-branes. Moreover, these $D6_{-\theta}$-branes become $D6_{-\frac{\pi}{2}}$-branes when they meet with the $NS5_R$-brane instantaneously and then after that they come back to the original $D6_{-\theta}$-branes. Therefore, in this dual process, there is no creation of D4-branes. Of course, the $D6_{-\theta}$-branes meet the $NS5'_R$-brane with an angle. Similarly, the N'_f $D6_{-\theta'}$-branes meet the $NS5'_L$-brane with *no* angles and the N''_f $D6_{-\theta''}$-branes meet the $NS5'_L$-brane and the $NS5_R$-brane with *no* angles. Then the dual color number $2\widetilde{N}_c$ is given by $2\widetilde{N}_c = 2N_f + 2N'_f + 2N''_f - 2N''_c - 4$.

The $NS5_R$-brane starts out with linking number $l_e = 2N''_c - 2N'_c + 2$ and after duality this $NS5_R$-brane ends up with linking number $l_m = 2\widetilde{N}'_c - 2\widetilde{N}_c - 2$. As we observed above, we consider only the particular brane motion where all the $D6_{-\theta,-\theta',-\theta''}$-branes become $D6_{-\frac{\pi}{2}}$-branes when they meet with the $NS5_R$-brane instantaneously and after that they come back to the original $D6_{-\theta,-\theta',-\theta''}$-branes. Therefore, in this dual process, there is *no* creation of D4-branes. Then it turns out that the dual color number $2\widetilde{N}'_c$ is given by $2\widetilde{N}'_c = 2N_f + 2N'_f + 2N''_f - 2N'_c + 4$.

The $NS5_L$-brane starts out with linking number $l_e = 2N_c - \frac{(2N'_f + 2N''_f)}{2} + 2$ and after duality

this $NS5_L$-brane ends up with linking number $l_m = -2\widetilde{N}_c'' + 2N_f + \frac{(2N_f'+2N_f'')}{2} - 2$. As we observed above, we consider only the particular brane motion where all the $D6_{-\theta',-\theta''}$-branes meet the $NS5_L$-brane *with angles.* Then it turns out that the dual color number $2\widetilde{N}_c''$ is given by $2\widetilde{N}_c'' = 2N_f + 2N_f' + 2N_f'' - 2N_c - 4$. Then one obtains the folllowing dual color numbers

$$
\begin{aligned}
2\widetilde{N}_c &= 2N_f + 2N_f' + 2N_f'' - 2N_c'' - 4, \\
2\widetilde{N}_c' &= 2N_f + 2N_f' + 2N_f'' - 2N_c' + 4, \\
2\widetilde{N}_c'' &= 2N_f + 2N_f' + 2N_f'' - 2N_c - 4.
\end{aligned}
$$

The low energy theory on the three color D4-branes has $Sp(\widetilde{N}_c)\times SO(2\widetilde{N}_c')\times Sp(\widetilde{N}_c'')$ gauge group and $2N_f$-fundamental dual quarks q'', $2N_f'$-vectors dual quarks q', $2N_f''$-fundamental dual quarks q, bifundamentals f, g and various gauge singlets. The f is in the representation $(\mathbf{2\widetilde{N}_c}, \mathbf{2\widetilde{N}_c'}, \mathbf{1})$ while the g is in the representation $(\mathbf{1}, \mathbf{2\widetilde{N}_c'}, \mathbf{2\widetilde{N}_c''})$, under the dual gauge group. The $2N_f''$ flavors q are in the representation $(\mathbf{2\widetilde{N}_c}, \mathbf{1}, \mathbf{1})$, the $2N_f'$ flavors q' are in the representation $(\mathbf{1}, \mathbf{2\widetilde{N}_c'}, \mathbf{1})$, and the $2N_f$ flavors q'' are in the representation $(\mathbf{1}, \mathbf{1}, \mathbf{2\widetilde{N}_c''})$, under the gauge group. In particular, a magnetic meson field $M_0 \equiv QQ$ is $2N_f \times 2N_f$ matrix and comes from 4-4 strings of $2N_f$ flavor D4-branes(when $2N_f$ $D6_{-\theta}$-branes meet the $NS5_R'$-brane), a magnetic meson field $M_0' \equiv Q'Q'$ is $2N_f' \times 2N_f'$ matrix and comes from 4-4 strings of $2N_f'$ flavor D4-branes(when $2N_f'$ $D6_{-\theta'}$-branes meet the $NS5_L$-brane) and a magnetic meson field $M_0'' \equiv Q''Q''$ is $2N_f'' \times 2N_f''$ matrix and comes from 4-4 strings of $2N_f''$ flavor D4-branes(when $2N_f''$ $D6_{-\theta''}$-branes meet the $NS5_L$-brane).

Then the most general magnetic superpotential, for the case where $2N_f(2N_f')[2N_f'']$ $D6_{-\theta}$-branes($D6_{-\theta'}$-branes)[$D6_{-\theta}$-branes] meet the NS-branes *with angles*, is given by

$$
\begin{aligned}
W_{dual} &= \left[f^4 + f^2g^2 + g^4\right] \\
&+ \left(\frac{\alpha}{2}\,\mathrm{tr}\, M_0^2 - mM_0\right) + \left(\frac{\alpha'}{2}\,\mathrm{tr}\, M_0'^2 - m'M_0'\right) + \left(\frac{\alpha''}{2}\,\mathrm{tr}\, M_0''^2 - m''M_0''\right) \\
&+ \left[M_0 q'' f^4 q'' + M_0' q' f^4 q' + M_0'' q f^4 q\right] \\
&+ \left[M_2 q'' f^2 q'' + M_4 q'' q'' + M_{2,F}' q' g^2 q' + M_{2,G}' q' f^2 q' + M_4' q' q' + M_2'' q f^2 q\right. \\
&+ \left.M_4'' qq + P_1 q' f^3 q'' + P_2 q f g q'' + P_3 q' f q'' + R_1 q' f^4 q + R_3 q' f q\right]
\end{aligned}
\tag{5.2}
$$

where the mesons are given by

$$
\begin{aligned}
M_0 &\equiv QQ, \quad M_0' \equiv Q'Q', \quad M_0'' \equiv Q''Q'', \quad M_2 \equiv QF^2Q, \quad M_4 \equiv QF^4Q, \\
M_{2,F}' &\equiv Q'F^2Q', \quad M_{2,G}' \equiv Q'G^2Q', \quad M_4' \equiv Q'F^4Q', \quad M_2'' \equiv Q''G^2Q'', \\
M_4'' &\equiv Q''G^4Q'', \quad P_1 \equiv QFQ', \quad P_2 \equiv QFGQ'', \quad P_3 \equiv QF^3Q', \\
R_1 &\equiv Q'GQ'', \quad R_3 \equiv Q'G^3Q''.
\end{aligned}
$$

The first two lines of (5.2) are dual expressions for the electric superpotential (5.1) and the corresponding meson fields M_0, M_0', M_0'' are replaced and the remaining lines of (5.2) are the analogs of the cubic term superpotential between the meson and dual quarks in Seiberg duality.

When the N_f $D6_{-\theta}$-branes meet the $NS5_L'$-brane and $NS5_R$-brane, *no* creation of D4-branes implies that there is no M_2- or M_4-term in the above superpotential (5.2)(and their mirrors). The mesons M_2 and M_4 originate from $Sp(N_c)$ chiral mesons QQ when one dualizes the $Sp(N_c)$ gauge group first by moving the $NS5_L'$-brane to the left of the $NS5_L$-brane. That is, the fluctuations of strings stretching between the $2N_f$ "flavor" D4-branes provide these meson fields. After the additional dual procedures, the cubic terms in the superpotential arise as M_2-dependent and M_4-dependent terms where M_2 has extra F^2 fields and M_4 has extra F^4 fields, besides QQ, due to the further dualizations. The M_2-term in the superpotential has an extra f^2 factor besides $q''q''$.

When the N_f'' $D6_{-\theta''}$-branes meet the $NS5_L'$-brane and $NS5_R$-brane, no creation of D4-branes implies that there is no M_2''- or M_4''-term in the above superpotential (5.2)(and their mirrors). The mesons M_2'' and M_4'' originate from $Sp(N_c'')$ chiral mesons $Q''Q''$ when one dualizes the $Sp(N_c'')$ gauge group first by moving the $NS5_R$-brane to the right of the $NS5_R'$-brane. That is, the fluctuations of strings stretching between the $2N_f''$ "flavor" D4-branes provide these meson fields. After the additional dual procedures, the cubic terms in the superpotential arise as M_2''-dependent and M_4''-dependent terms where M_2'' has extra G^2 fields and M_4'' has extra G^4 fields, besides $Q''Q''$, due to the further dualizations. The M_2''-term in the superpotential has an extra f^2 factor besides qq.

Similarly, when the N_f' $D6_{-\theta'}$-branes meet the $NS5_L'$-brane, the $NS5_R$-brane, or $NS5_R'$-brane with no angles, there is no M_4' term in the above superpotential (5.2)(and their mirrors) [2]. These meson fields originate from $SO(2N_c')$ chiral mesons $Q'Q'$ when one dualizes the $SO(N_c')$ gauge group first by interchanging the $NS5_L'$-brane and the $NS5_R$-brane each other. The strings stretching between the $2N_f'$ "flavor" D4-branes provide this meson. After the additional dual procedures, the cubic terms in the superpotential arise as $M_{2,F}'$-term, $M_{2,G}'$-term and M_4'-term in (5.2) where $M_{2,F}'$ has extra F^2 fields, $M_{2,G}'$ has extra G^2 fields, and M_4' has extra F^4 fields, besides $Q'Q'$, due to the further dualizations. The $M_{2,F}'$-term in the superpotential has extra g^2 factor while $M_{2,G}'$-term has extra f^2 dependent factor, besides

[2] In general, $4N_f'$ full D4-branes without changing the linking number should be added in order to satisfy the correct dual color numbers in the gauge theory side analysis. This has led to the fact that there are $4N_f'$ D4-branes connecting the $NS5_R$-brane and N_f' $D6_{-\theta'}$-branes, after duality. In other words, these extra $4N_f'$ D4-branes were needed for the existence of meson fields $M_{2,F}'$ and $M_{2,G}'$. In our construction, we do not need these extra $4N_f'$ full D4-branes because we do not want to have these unwanted meson fields $M_{2,F}'$ and $M_{2,G}'$.

$q'q'$.

Furthermore, when the N_f $D6_{-\theta}$-branes, the N'_f $D6_{-\theta'}$-branes, the $NS5'_L$-brane and the $NS5_R$-brane meet each other with no angles, no P_1- and P_3-dependent terms arise in the superpotential (5.2). These mesons originate from $SO(2N'_c)$ chiral mesons FQ' when one dualizes the $SO(2N'_c)$ first by interchanging the $NS5'_L$-brane the $NS5_R$-brane. The strings stretching between the $2N'_f$ flavor D4-branes and $2N_c$ color D4-branes give rise to these $2N'_f$ $Sp(N_c)$ fundamentals. After the additional dual procedures, these cubic terms arise as these meson terms where there exist extra f^2q'' and q'' of interactions in P_1 and P_3 in the superpotential and these mesons have extra Q, QF^2 fields respectively, due to the further dualizations.

When the N'_f $D6_{-\theta'}$-branes, the N''_f $D6_{-\theta''}$-branes, the $NS5'_L$-brane and the $NS5_R$-brane meet each other with no angles, no R_1- and R_3-dependent terms arise in the superpotential (5.2). These mesons originate from $Sp(N''_c)$ chiral mesons GQ'' when one dualizes the $Sp(N''_c)$ first by moving the $NS5_R$-brane to the right of the $NS5'_R$-brane. The strings stretching between the $2N''_f$ flavor D4-branes and $2N'_c$ color D4-branes give rise to these $2N''_f$ $SO(2N'_c)$ vectors. After the additional dual procedures, these cubic terms arise as these meson terms where R_1 and R_3 have extra $Q', Q'G^2$ fields respectively, due to the further dualizations.

Finally, when the N_f $D6_{-\theta}$-branes, the N''_f $D6_{-\theta''}$-branes, the $NS5'_L$-brane and the $NS5_R$-brane meet each other with no angles, no P_2-dependent term occurs in the superpotential (5.2). These mesons originate from $Sp(N_c)$ chiral mesons QF when one dualizes the $Sp(N_c)$ first by moving the $NS5'_L$-brane to the left of the $NS5_L$-brane. The strings stretching between the $2N_f$ flavor D4-branes and $2N'_c$ color D4-branes give rise to these $2N_f$ $SO(2N'_c)$ vectors. After the additional dual procedures, these cubic terms arise as these meson terms where there exist extra gq'' in the P_2 interaction term and this meson has extra GQ'' fields respectively, due to the further dualizations.

Then the reduced magnetic superpotential in our case by taking the first three lines of (5.2) is given by

$$\begin{aligned} W_{dual} &= \left[M_0 q'' f^4 q'' + \frac{\alpha}{2} \operatorname{tr} M_0^2 - m M_0\right] + \left[M'_0 q' f^4 q' + \frac{\alpha'}{2} \operatorname{tr} {M'_0}^2 - m' M'_0\right] \\ &+ \left[M''_0 q f^4 q + \frac{\alpha''}{2} \operatorname{tr} {M''_0}^2 - m'' M''_0\right]. \end{aligned} \tag{5.3}$$

For the supersymmetric vacua, one can compute the F-term equations for this superpo-

tential (5.3) and the F-terms for $M_0, q'', M'_0, q', M''_0, q$ and f are given by

$$
\begin{aligned}
& q'' f^4 q'' - m + \alpha M_0 = 0, \qquad (M_0 q'' f^2) f^2 + f^2 (f^2 q'' M_0) = 0, \\
& q' f^4 q' - m' + \alpha' M'_0 = 0, \qquad (M'_0 q' f^2) f^2 + f^2 (f^2 q' M'_0) = 0, \\
& q f^4 q - m'' + \alpha'' M''_0 = 0, \qquad (M''_0 q f^2) f^2 + f^2 (f^2 q M''_0) = 0, \\
& f(f^2 q'' M_0) q'' + f q'' (M_0 q'' f^2) + f(f^2 q' M'_0) q' + f q' (M'_0 q' f^2) + f(f^2 q M''_0) q \\
& + f q (M''_0 q f^2) + (f^2 q'' M_0) q'' f + q'' (M_0 q'' f^2) f + (f^2 q' M'_0) q' f + q' (M'_0 q' f^2) f \\
& + (f^2 q M''_0) q f + q' (M''_0 q f^2) f = 0.
\end{aligned}
$$

From this, it is easy to see that the last equation is satisfied if the second, fourth and sixth are satisfied:$M_0 q'' f^2 = 0 = \cdots = f^2 q M''_0$.

The theory has many nonsupersymmetric meta-stable ground states and when we rescale the meson fields as $M_0 = h\Lambda\Phi_0, M'_0 = h'\Lambda'\Phi'_0$ and $M''_0 = h''\Lambda''\Phi''_0$, then the Kahler potential for Φ_0, Φ'_0 and Φ''_0 is canonical and the magnetic quarks are canonical near the origin of field space. Then the magnetic superpotential (5.3) can be rewritten as

$$
\begin{aligned}
W_{mag} &= \left[h\Phi_0 q'' f^4 q'' + \frac{h^2 \mu_\phi}{2} \operatorname{tr} \Phi_0^2 - h\mu^2 \operatorname{tr} \Phi_0 \right] + \left[h'\Phi'_0 q' f^4 q' + \frac{h'^2 \mu'_\phi}{2} \operatorname{tr} {\Phi'_0}^2 - h'\mu'^2 \operatorname{tr} \Phi'_0 \right] \\
&+ \left[h''\Phi''_0 q f^4 q + \frac{h''^2 \mu''_\phi}{2} \operatorname{tr} \Phi''^2_0 - h''\mu''^2 \operatorname{tr} \Phi''_0 \right]
\end{aligned}
$$

where $\mu^2 = m\Lambda, \mu'^2 = m'\Lambda', \mu''^2 = m''\Lambda''$ and $\mu_\phi = \alpha\Lambda^2, \mu'_\phi = \alpha'\Lambda'^2, \mu''_\phi = \alpha''\Lambda''^2$.

Now one splits the $2(N_f - l) \times 2(N_f - l)$ block at the lower right corner of $h\Phi_0$ and $q'' f^4 q''$ into blocks of size $2n$ and $2(N_f - l - n)$, one decomposes the $2(N'_f - l') \times 2(N'_f - l')$ block at the lower right corner of $h'\Phi'_0$ and $q' f^4 q'$ into blocks of size $2n'$ and $2(N'_f - l' - n')$ and one splits the $2(N''_f - l'') \times 2(N''_f - l'')$ block at the lower right corner of $h''\Phi''_0$ and $q f^4 q$ into blocks of size $2n''$ and $2(N''_f - l'' - n'')$ as follows:

$$
\begin{aligned}
h\Phi_0 &= \begin{pmatrix} 0_{2l} & 0 & 0 \\ 0 & h\Phi_{2n} & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - l - n} \otimes i\sigma_2 \end{pmatrix}, h'\Phi'_0 = \begin{pmatrix} 0_{2l'} & 0 & 0 \\ 0 & h'\Phi'_{2n'} & 0 \\ 0 & 0 & \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{N'_f - l' - n'} \otimes \sigma_3 \end{pmatrix}, \\
h''\Phi''_0 &= \begin{pmatrix} 0_{2l''} & 0 & 0 \\ 0 & h''\Phi''_{2n''} & 0 \\ 0 & 0 & \frac{\mu''^2}{\mu''_\phi} \mathbf{1}_{N''_f - l'' - n''} \otimes i\sigma_2 \end{pmatrix}, q'' f^4 q'' = \begin{pmatrix} \mu^2 \mathbf{1}_{2l} & 0 & 0 \\ 0 & \varphi'' y^4 \varphi'' & 0 \\ 0 & 0 & 0_{2(N_f - l - n)} \end{pmatrix}, \\
q' f^4 q' &= \begin{pmatrix} \mu'^2 \mathbf{1}_{2l'} & 0 & 0 \\ 0 & \varphi' y^4 \varphi' & 0 \\ 0 & 0 & 0_{2(N'_f - l' - n')} \end{pmatrix}, q f^4 q = \begin{pmatrix} \mu''^2 \mathbf{1}_{2l''} & 0 & 0 \\ 0 & \varphi y^4 \varphi & 0 \\ 0 & 0 & 0_{2(N''_f - l'' - n'')} \end{pmatrix}. \quad (5.4)
\end{aligned}
$$

Here φ'' is $2n \times 2(\widetilde{N}_c''-l)$ dimensional matrices, φ' is $2n' \times 2(\widetilde{N}_c'-l')$ dimensional matrices and φ is $2n'' \times 2(\widetilde{N}_c-l'')$ dimensional matrices. The φ'' corresponds to fundamental strings connecting the $2n$ flavor D4-branes and $2(\widetilde{N}_c''-l)$ color D4-branes, φ' corresponds to fundamental strings connecting the $2n'$ flavor D4-branes and $2(\widetilde{N}_c'-l')$ color D4-branes and φ corresponds to fundamental strings connecting the $2n''$ flavor D4-branes and $2(\widetilde{N}_c-l'')$ color D4-branes. The Φ_{2n} and $\varphi'' y^4 \varphi''$ are $2n \times 2n$ matrices, $\Phi'_{2n'}$ and $\varphi' y^4 \varphi'$ are $2n' \times 2n'$ matrices and $\Phi''_{2n''}$ and $\varphi y^4 \varphi$ are $2n'' \times 2n''$ matrices.

The supersymmetric ground state corresponds to $h\Phi_{2n} = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n \otimes i\sigma_2, \varphi'' y^2 = 0 = y^2 \varphi''$, $h'\Phi'_{2n'} = \frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{n'} \otimes \sigma_3, \varphi' y^2 = 0 = y^2 \varphi'$ and $h''\Phi''_{2n''} = \frac{\mu''^2}{\mu''_\phi} \mathbf{1}_{n''} \otimes i\sigma_2, \varphi y^2 = 0 = y^2 \varphi$. The l of the upper N_f-flavor D4-branes are reconnected with l-color D4-branes and the resulting l D4-branes stretch from the upper $D6_{-\theta}$-branes to the $NS5_L$-brane directly and the intersection point between the l D4-branes and the $D6_{-\theta}$-branes is given by $(v, w) = (+v_{D6_{-\theta}}, 0)$. The mirrors are located at $(v, w) = (-v_{D6_{-\theta}}, 0)$. This corresponds to exactly the $2l$'s eigenvalues from zeros of $h\Phi_0$ in (5.4). Now the remaining upper $(N_f - l)$-flavor D4-branes between the upper $D6_{-\theta}$-branes and the $NS5'_L$-brane correspond to the eigenvalues of $h\Phi_0$ in (5.4), i.e., $\frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - l}$. The intersection point between the upper $(N_f - l)$ D4-branes and the $NS5'_L$-branes is given by $(v, w) = (0, +v_{D6_{-\theta}} \cot\theta)$ from trigonometric geometry. The mirrors are located at $(v, w) = (0, -v_{D6_{-\theta}} \cot\theta)$.

Similarly, the l' of the upper N'_f-flavor D4-branes are reconnected with l'-color D4-branes and the resulting l' D4-branes stretch from the upper $D6_{-\theta'}$-branes to the $NS5_R$-brane directly and the intersection point between the l' D4-branes and the upper $D6_{-\theta'}$-branes is given by $(v, w) = (+v_{D6_{-\theta'}}, 0)$. The mirrors are located at $(v, w) = (-v_{D6_{-\theta'}}, 0)$. This corresponds to exactly the $2l'$'s eigenvalues from zeros of $h'\Phi'_0$ in (5.4). Now the remaining upper $(N'_f - l')$-flavor D4-branes between the upper $D6_{-\theta'}$-branes and the $NS5'_R$-brane correspond to the eigenvalues of $h'\Phi'_0$ in (5.4), i.e., $\frac{\mu'^2}{\mu'_\phi} \mathbf{1}_{N'_f - l'}$. The intersection point between the upper $(N'_f - l')$ D4-branes and the $NS5'_R$-branes is given by $(v, w) = (0, +v_{D6_{-\theta'}} \cot\theta')$ from trigonometric geometry. The mirrors are located at $(v, w) = (0, -v_{D6_{-\theta'}} \cot\theta')$.

The l'' of the upper N''_f-flavor D4-branes are reconnected with l''-color D4-branes and the resulting l'' D4-branes stretch from the upper $D6_{-\theta''}$-branes to the $NS5_R$-brane directly and the intersection point between the l'' D4-branes and the upper $D6_{-\theta''}$-branes is given by $(v, w) = (+v_{D6_{-\theta''}}, 0)$. The mirrors are located at $(v, w) = (-v_{D6_{-\theta''}}, 0)$. This corresponds to exactly the $2l''$'s eigenvalues from zeros of $h''\Phi''_0$ in (5.4). Now the remaining upper $(N''_f - l'')$-flavor D4-branes between the upper $D6_{-\theta''}$-branes and the $NS5'_R$-brane correspond to the eigenvalues of $h''\Phi''_0$ in (5.4), i.e., $\frac{\mu''^2}{\mu''_\phi} \mathbf{1}_{N''_f - l''}$. The intersection point between the upper $(N''_f - l'')$ D4-branes and the $NS5'_R$-branes is given by $(v, w) = (0, +v_{D6_{-\theta''}} \cot\theta'')$ from trigonometric

geometry. The mirrors are located at $(v, w) = (0, -v_{D6_{-\theta''}} \cot\theta'')$.

Now the full one loop potential containing $\Phi_{2n}, \Phi'_{2n'}, \Phi''_{2n''}$, by combining the superpotential and the vacuum expectation values for the fields, takes the form

$$\begin{aligned} V &= |h\Phi_{2n}\varphi'' y^2|^2 + |hy^2\varphi''\Phi_{2n}|^2 + |h\varphi'' y^4 \varphi'' - h\mu^2 \mathbf{1}_{2n} + h^2\mu_\phi \Phi_{2n}|^2 \\ &+ |h'\Phi'_{2n'}\varphi' y^2|^2 + |h'y^2\varphi'\Phi'_{2n'}|^2 + |h'\varphi' y^4\varphi' - h'\mu'^2\mathbf{1}_{2n'} + h'^2\mu'_\phi\Phi'_{2n'}|^2 \\ &+ |h''\Phi''_{2n''}\varphi y^2|^2 + |h''y^2\varphi\Phi''_{2n''}|^2 + |h''\varphi y^4\varphi - h''\mu''^2\mathbf{1}_{2n''} + h''^2\mu''_\phi\Phi''_{2n''}|^2 \\ &+ b|h^2\mu|^2 \,\mathrm{tr}\, \Phi_{2n}\Phi_{2n} + b'|h'^2\mu'|^2 \,\mathrm{tr}\, \Phi'_{2n'}\Phi'_{2n'} + b''|h''^2\mu''|^2 \,\mathrm{tr}\, \Phi''_{2n''}\Phi''_{2n''} \end{aligned}$$

where $b = \frac{(\ln 4-1)}{8\pi^2}\widetilde{N}_c$, $b' = \frac{(\ln 4-1)}{8\pi^2}\widetilde{N}'_c$ and $b'' = \frac{(\ln 4-1)}{8\pi^2}\widetilde{N}''_c$. Differentiating this potential with respect to Φ_{2n}, $\Phi'_{2n'}$ and $\Phi''_{2n''}$ and putting $\varphi'' y^2 = 0 = y^2\varphi''$, $\varphi' y^2 = 0 = y^2\varphi'$ and $\varphi y^2 = 0 = y^2\varphi$, one obtains

$$\begin{aligned} h\Phi_{2n} &\simeq \frac{\mu_\phi}{b}\mathbf{1}_n \otimes i\sigma_2 \qquad \text{or} \qquad M_{2n} \simeq \frac{\alpha\Lambda^3}{\widetilde{N}_c}\mathbf{1}_n \otimes i\sigma_2, \\ h'\Phi'_{2n'} &\simeq \frac{\mu'_\phi}{b'}\mathbf{1}_{n'} \otimes \sigma_3 \qquad \text{or} \qquad M'_{2n'} \simeq \frac{\alpha'\Lambda'^3}{\widetilde{N}'_c}\mathbf{1}_{n'} \otimes \sigma_3, \\ h''\Phi''_{2n''} &\simeq \frac{\mu''_\phi}{b''}\mathbf{1}_{n''} \otimes i\sigma_2 \qquad \text{or} \qquad M''_{2n''} \simeq \frac{\alpha''\Lambda''^3}{\widetilde{N}''_c}\mathbf{1}_{n''} \otimes i\sigma_2, \end{aligned}$$

corresponding to the w coordinates of $2n$ curved flavor D4-branes between the $D6_{-\theta}$-branes and the $NS5'_L$-brane, the w coordinates of $2n'$ curved flavor D4-branes between the $D6_{-\theta'}$-branes and the $NS5'_R$-brane, the w coordinates of $2n''$ curved flavor D4-branes between the $D6_{-\theta''}$-branes and the $NS5'_R$-brane respectively.

5.3 Addition of adjoint fields

When we consider higher order superpotential for the adjoint fields whose degree is greater than four(i.e., the number of NS5-branes or NS5'-branes $k > 3$), there exist more meson fields. Since it is not known how the deformations for the meson fields QX_1^jQ, QX_2^jQ or $Q''X_3^jQ''$ arise geometrically in the type IIA brane configuration, it is not clear how to add these deformations in an electric theory or a magnetic theory.

5.4 $SO(2N_c) \times Sp(N'_c) \times SO(2N''_c)$ with $2N_f$-vectors, $2N'_f$-fund., $2N''_f$-vectors, and bifund.

In this case, compared to the previous case, the charge of orientifold 4-plane is reversed. Correspondingly, the orthogonal(symplectic) gauge group is changed into symplectic(orthogonal)

gauge group. The only difference appears in the symmetric and antisymmetric property for the meson field and adjoint field. We do not present this here but it can be done by following the prescription of previous section.

6 Conclusions and outlook

We have found the type IIA nonsupersymmetric meta-stable brane configurations for two gauge group theory with fundamentals, bifundamentals and adjoints, three gauge group theory with fundamentals and bifundamentals, and their orientifold 4-plane generalizations. It would be interesting to discover whether the meta-stable brane configurations exist for the theory with no D6-branes [34, 35, 36, 37] when we take the dual for the whole gauge groups. For the three gauge group theory with fundamentals and bifundamentals, it is also possible to take the dual only for two gauge groups among three. Then it would be interesting to see how the meta-stable brane configurations occur.

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